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EFFECTS OF LOGICAL FORM AND GEOMETRICAL CONTENT ON SPATIAL DEDUCTIVE REASONING

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Executive Summary

In air navigation, pilots must mentally represent the relative position of aircraft in three dimensions. They must also deduce from their mental representation the relative position of other aircraft which has not been explicitly specified in any of the information that they have received. Spatial deductive reasoning is critical for both the pilot's safety and the accomplishment of his mission. It is a difficult aspect of formal logic which is particularly prone to errors, especially under intense mental workload. Despite the importance of spatial deductive reasoning, the processes that underlie this logical activity are still unclear as scientists have proposed opponent theories of formal logic to account for these processes.

The overall goal of this study is to investigate the effects of logical form and geometrical content on spatial deductive reasoning by comparing two opponent theories of deductive reasoning: Hagert's Formal Rules theory and Johnson-Laird's Mental Models theory. Our second goal is to specify, through the effects of geometrical content, how humans structure their mental representation of geometrical relations, and if they do so relative to spatial reference frames. We will address this issue in view of the Content-Specific Rules theory and the Spatial Reference Frame theory.

Twenty-six subjects solved 144 spatial deductive problems which differed by their logical form and geometrical content. We addressed the effects of logical form by varying the continuity and the determinacy of the entities' order in the arguments. The logical form of the arguments allowed comparison of pairs of problems having either (a) formal derivations of equal length but different numbers of mental models, and (b) formal derivations of different lengths but equal numbers of mental models. We addressed the effects of geometrical content by varying the euclidian (number of dimensions) and projective relations (orientation and direction) among the entities. The number of mental models and the number of steps in the formal derivations of valid conclusions were the same across geometrical contents. Overall, the effects of logical form and geometrical content confirmed the Mental Models theory's predictions while refuting those of the Formal Rules theory. The effects of geometrical content suggest that subjects constructed mental models which reproduce the geometrical relations among entities relative to spatial reference frames. This study will provide an experimental basis from which to measure spatial reasoning in pilots, and help design visualization techniques to train air-to-air basic flight maneuvers.

Abstract

This study aims to elucidate the effects of the logical form and geometrical content of an argument on spatial deductive reasoning. We will investigate the effects of these factors by comparing the opposing predictions made by the Formal Rules theory and the Mental Models theory of spatial deductive reasoning. We will also elucidate the effects of geometrical content in view of the Content-Specific Rules theory and the Spatial Reference Frame theory to specify whether humans construct their mental representations relative to spatial reference frames, and how these affect the ease of processing geometrical relations.

Twenty-six subjects solved 144 spatial deductive problems which differed by their logical form and geometrical content. We addressed the effects of logical form by varying the continuity and the determinacy of the entities' order in the arguments to determine the relative importance of the two factors on the processes of deduction. The logical form of the arguments allowed comparison of pairs of problems having either (a) formal derivations of equal length but different numbers of mental models, and (b) formal derivations of different lengths but equal numbers of mental models. We addressed the effects of geometrical content by varying the euclidian (number of dimensions) and projective relations (orientation and direction) among the entities. The number of mental models and the number of steps in the formal derivations of valid conclusions were the same across geometrical contents.

The effects of referential continuity showed that deductions from the semicontinuous order were significantly easier than from the discontinuous order although the former required longer formal derivations than the latter. In the semicontinuous order, problems based on two mental models were significantly more difficult than problems based on one mental model despite the fact that the former involved a shorter formal derivation than the latter. The results generalized to spatial reasoning in all three dimensions. In the discontinuous order, problem types involving the representation of two independent mental layouts (i.e., partial mental models) were significantly more difficult than those involving the representation of two related mental layouts, although the problem types involved formal derivations of the same length. The effects of logical form confirmed the Mental Models theory's predictions while refuting those of the Formal Rules theory. The results suggest that referential discontinuity has precedence over referential indeterminacy in determining the difficulty of spatial reasoning.

The difficulty of spatial reasoning varied reliably with the geometrical content of the problems although they involved identical formal derivations. Hence, the difficulty of spatial reasoning increased systematically with the number of dimensions (1D, 2D, and 3D) that subjects had to integrate and inspect within a mental model. The difficulty of reasoning in 3D was systematically more difficult from vertical layouts than from horizontal ones. Subjects also showed systematic patterns of directional preferences in constructing mental models from one set of directions rather than from opposite ones. The effects of geometrical content suggest that subjects construct mental models which reproduce the geometrical relations among entities relative to spatial reference frames.

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In air, marine, or ground navigation, military officers are required to represent and deduce the relative location of entities in three dimensions. For instance, given the information that A is to the left of B which is above C and C is in front of D[1], and a question concerning the relative location of A and D; one may deduce that A is to the left, above, and in front of D. Such deductions are an intrinsic part of human cognitive activity.

The cognitive process by which humans represent and reason about relative locations is called spatial deductive reasoning. It is an aspect of logic which may be affected by the geometrical content of a problem and/or its logical form. The geometrical content, which is of interest in this study, specifies the relative location of entities according to euclidean (e.g., number of dimensions), projective (e.g., orientation, direction) and topological (e.g., proximity, enclosure) relations. These geometrical relations are deictic in that they specify the location of an object relative to a reference object using projective prepositions, such as *left of, in front of,* or *above.* Consider the following argument (premises and conclusion):

Star (B) left of circle (A); square (C) left of star (B); bar (D) directly left of star (B); square (C) directly right of cross (E). Is cross (E) right of bar (D)?

The premises describe the layouts of five entities, from right to left, in one horizontal dimension. The reference object (A) specifies the origin of the axis. The resulting layout of the entities is illustrated below.

ECDBA

$\longleftarrow$

Once abstracted from its geometrical content, the above argument has the following logical form:

B r A; C r B; D r B; C r E. Is E r D?

where A, B, C, D, and E are variables that arbitrarily denote one of the above five entities, and r represents a transitive relation between two variables ordered according to the sequence BA, CB, DB, CE. The logical form of a premise, such as B r A, denotes a proposition which has a truth value independent of its content.

While geometrical content specifies relations among entities, logical form specifies order among the entities. The logical form of an argument can vary according to different levels of referential continuity and referential determinacy. Referential continuity specifies the extent to which adjacent pairs of premises have a common referent (or entity). In the following example --A r B; B r C; C r D; D r E-- the order of the referents is continuous. Referential determinacy specifies the extent to which the order of referents is determinate or unambiguous. In the following example --B directly left of A; C directly left of B-- the order of the referents is determinate in that each entity occupies a unique position relative to another, thus yielding one layout, namely CBA.

1 In this example, as in the following ones, we will assume that the entities are viewed from a fronto-parallel perspective and that they are collinear on a given axis.

Cognitive scientists have proposed three main theoretical views to account for the role of logical form and geometrical content on spatial deductive reasoning. The first view (Hagert, 1985; Hagert & Hansson, 1983, 1984) assumes formal rules of inference applied strictly to the logical form of an argument. The second posits content-specific rules of inference (DeSoto, London, & Handel, 1965; Huttenlocher, 1968). The third view (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991, 1993) argues semantic procedures that construct mental models of the premises based on their logical form and geometrical content. The role of logical form and geometrical content on spatial deductive reasoning is thus still a matter of theoretical controversy. Consequently, analysis of the effects of logical form and geometrical content should elucidate the processes that underlie spatial deductive reasoning, and thus help decide between the above theoretical views of spatial reasoning.

The overall goal of this study is to investigate the effects of logical form and geometrical content on spatial deductive reasoning. We will assess the effects of logical form by varying the continuity and the determinacy of the entities' order in the arguments; those of geometrical content by varying the euclidian (number of dimensions) and projective (orientation and direction) relations among the entities. We will investigate the effects of these variables in light of the above three theoretical views of spatial deductive reasoning. Our second goal is to specify, through the effects of the geometrical content, how humans structure their mental representation of geometrical relations, and if they do so relative to spatial reference frames. Although the concept of spatial reference frames has been applied to the perception and representation of deictic spatial relations (Logan, 1995), its use in the area of deductive reasoning has not yet been investigated. Consequently, analysis of the effects of geometrical content will help specify whether the Spatial Reference Frame theory (Logan, 1995) can be extended to spatial deductive reasoning while providing a complementary account of the above three theoretical views. We will now consider all four views.

1. Theories of Spatial Deductive Reasoning

1.1 Formal Rules Theories

Formal Rules theorists argue that humans use formal rules of inference, such as the rule of *transitivity*, to derive conclusions from the logical form of a set of premises irrespective of their content (Braine, 1978; Braine & O'Brien, 1991; Hagert, 1985; Piaget, 1972). A sequence of formal rules of inference constitutes a formal derivation of the conclusion from the premises (Rips, 1983).

In the simplest spatial deductive problem, a formal derivation has a single inferential step formed by applying just one formal rule, namely the rule of transitivity: *if A r B and B r C, then A r C.* In more complex spatial deductive problems, humans will require more than a single inferential step to derive a conclusion from the premises (Byrne & Johnson-Laird, 1989; Hagert, 1985; Hagert & Hansson, 1983, 1984). But in all cases, the difficulty of a spatial deductive problem depends on the length of the formal derivation underlying its solution. In turn, the length of a formal derivation depends purely on the logical form of an argument, that is, the order of the objects in the premises and conclusion(s). This factor is critical in determining working memory load, and thus the difficulty of deductive reasoning (Braine & O'Brien, 1991; Rips, 1983). In principle, the geometrical relations described among objects, such as number of dimensions or directions, should not affect the difficulty of spatial deductive reasoning since these factors pertain to the content of an argument. If such effects occur, Formal Rules theorists would consider them as extra-logical effects that may have their influence during the representation of the premises but not during the process of deduction itself (Braine, 1978; Hagert, 1985, Piaget, 1972; Rips, 1983).

1.2 Content-Specific Rules Theories

Content-Specific Rules theorists (namely, DeSoto et al., 1965; Huttenlocher, 1968) have partly addressed content factors related to relational reasoning using basic transitive reasoning problems involving three terms (e.g., A is faster than B; B is faster than C. Who is the fastest?). DeSoto et al. (1965) argue that information described in a transitive problem is integrated in a spatial array, such as A B C, that depicts the serial ordering of three terms, as in the premises "A is faster than B; B is faster than C". The orientation and direction from which humans construct spatial arrays depend on the premises' relations. For example, certain relations, such as *taller*, lead to vertical arrays constructed from top to bottom, while opposite relations, such as *shorter*, lead to vertical arrays constructed from bottom to top. The difficulty of representing a spatial array in one orientation depends on the direction from which it is constructed. Spatial arrays constructed from top to bottom, or left to right, would be easier than those constructed from the opposite directions.

DeSoto et al. (1965) suggest that humans have *directional preferences* in building spatial arrays of entities along a particular axis, and from one direction rather than from the opposite one, presumably because of culturally-based reading patterns and the meaning of certain transitive relations such as *faster, taller*. Related studies (Handel, DeSoto, & London, 1968; Huttenlocher, 1968; Jones, 1970) have confirmed DeSoto et al.'s directional preference hypothesis. However, these studies have assessed the effects of direction for basic transitive problems involving nonspatial relations. Experimenters have yet to elucidate the effects of direction and other geometrical relations on more complex deductive problems involving spatial relations. The directional preference hypothesis also suggests that humans may be constructing spatial arrays relative to spatial reference frames which differentially affect the ease of processing spatial relations (Logan, 1995).

While the above issues remain to be investigated, Content-Specific Rules theories provide a complementary account of the Formal Rules theories in that they have shown that the content of a problem can affect deductive reasoning. But humans have the logical capacity to make deductions that depend on the logical form of an argument. In this respect, Content-Specific Rules theories are incomplete, as they do not account for the fundamental processes that underlie spatial deductive reasoning.

1.3 Mental Models Theory

The Mental Models theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991, 1993) assumes that spatial deductive reasoning is achieved through semantic procedures that construct and validate mental models of the premises. Humans draw valid conclusions by inspecting these mental models without resorting to formal rules of inference, or even content-specific rules of inference (Cheng & Holyoak, 1985; DeSoto et al., 1965).

The Mental Models theory does concede that the logical form and the geometrical content of argument affect the ease with which humans construct mental models and, thus, the difficulty of spatial deductive reasoning. The Mental Models theorists elucidated the effects of logical form by varying the continuity of the object order in the premises (Ehrlich & Johnson-Laird, 1982) and the determinacy of the object orders (Byrne & Johnson-Laird, 1989; Mani & Johnson-Laird, 1982). Byrne & Johnson-Laird (1989) investigated the effects of geometrical content by varying the number of dimensions (1D and 2D) described in the problems' premise sets. The above variables were used to compare the opposing predictions made the Formal Rules theory and the Mental Models theory concerning the representation (Ehrlich & Johnson-Laird, 1982; Mani & Johnson-Laird, 1982) or processes (Byrne & Johnson-Laird, 1989) of spatial deductive reasoning.

1.3.1 Role of Logical Form

1.3.1.1 Role of Referential Continuity. Ehrlich and Johnson-Laird (1982) elucidated the effects of referential continuity on the *representation* of 2D spatial relations by presenting descriptions of such relations in continuous, semicontinuous, and discontinuous orders. In the continuous order, each adjacent pair of a set of three sentences had a referent (or an object) in common; in the semicontinuous order, the second and third sentences had no referent in common; while in the discontinuous order, the first and second sentences had no referential continuity[2].

The authors made two alternative predictions. First, if subjects construct mental models, continuous and semicontinuous orders would be equally easy to represent as both orders allow the referents to be continuously integrated within a mental model. However, in the discontinuous order, the first two sentences would oblige the subjects to hold two independent mental layouts[3] in working memory before they could use the third sentence to integrate these mental layouts into a mental model. Thus, the third order would be the hardest of all.

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[2] Examples of the oral sentences are, for the first order: A is in the front of B, B is on the left of C, C is behind D; and for the third order: C is behind D, A is in the front of B, B is on the left of C. Single syllable words (knife, fork, spoon, glass) denoted the entities A to D.

[3] The term mental layout will signify a mental model that is being constructed.

Alternatively, if subjects construct mental propositions, as argued by the Formal Rules theories, the continuous order would be the easiest to represent because each successive sentence has a common referent. However, the semicontinuous order would be as difficult to represent as the discontinuous one as in both cases the order of the sentences is not optimal for establishing coreferential links. The results confirmed the Mental Models theory's prediction: continuous and semicontinuous orders were of equal ease, and both were reliably easier than the discontinuous order. These results thus suggest that humans construct mental models of 2D spatial relations.

Would the effects of referential continuity generalise to the *processes* that underlie spatial deductive reasoning in all three dimensions? This issue has not been addressed, and yet it remains a matter of theoretical controversy as Hagert's (1985) Formal Rules theory and Johnson-Laird's Mental Models theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991, 1993) make opposite predictions regarding the effects of referential continuity on the process of deduction. Consider, for example, the following two premise sets, one which describes the entities in a semicontinuous order, and the other which describes the entities in a discontinuous order:

Semicontinuous order: A right of B; C left of B; D directly left of B; C directly right of E. Is E left of D?

Discontinuous order: E directly left of C; B directly right of D; C left of B; A right of B. Is E left of D?

In the semicontinuous order, the third and fourth premise have no referent in common while in the discontinuous order, the first two premises have no referential continuity. In the semicontinuous order, the formal derivation of the location of E relative to D involves four inferential steps[4]; while in the discontinuous order, it involves only two inferential steps. The difference is due to the two operations of inversion required in the semicontinuous order [right(CE) <=> left(EC), and left(DB) <=> right(BD)] which are not in the discontinuous order. Thus, the Formal Rules theory would predict that deductions from the semicontinuous order should be more difficult than from the discontinuous order.

In contrast, the Mental Models theory predicts that mental models should be easier to construct from the semicontinuous order because each premise allows the continuous construction of a mental model. However, the first two premises of the discontinuous order require two independent mental layouts [EC and BD] to be held in working memory before their integration using the third premise. The effect of referential continuity on the processes of spatial reasoning thus yields opposite predictions from the two theories; however, these predictions remain to be tested.

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4 Using first order predicate logic, the inferential steps involved by the semicontinuous order are:

1- right(CE) <=> left(EC); 2- left(EC) & left(CB) => left(EB); 3- left(DB) <=> right(BD); 4- left(EB) & right(BD) => left(ED).

In the discontinuous order, the inferential steps are:

1- left(EC) & left(CB) => left(EB); 2- left(EB) & right(BD) => left(ED).

1.3.1.2 Role of Referential Determinacy. We have seen so far that referential continuity among premises affects the construction of an integrated mental representation, and should also affect the process of deduction itself. Referential determinacy would affect the total number of mental models consistent with the meaning of the premises and the length of a formal derivation. The Mental Models theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991) predicts that problems which involve multiple mental models should be more difficult than those which involve only one mental model. In contrast, Hagert's (1985) Formal Rules theory predicts that if the former problems involve a shorter a formal derivation, then they should be easier than the latter.

Byrne and Johnson-Laird (1989) tested these opposing predictions using three types (1, 2, and 3) of problems. Problems of type 1 and type 3 specified determinate orders between a set of entities, thus yielding a single mental model. However, problems of type 3 involved a longer formal derivation than those of type 1. Problems of type 2 described indeterminate orders between a set of entities, thus yielding multiple mental models. However, they involved the same formal derivation as those of type 1. All three problem types supported a valid conclusion.

Subjects were reliably more accurate in solving problems of type 3 based on one mental model, than problems of type 2 based on multiple mental models, although the former involved a longer formal derivation than the latter. The results thus confirmed the Mental Models theory's principle prediction while refuting that of Hagert's Formal Rules theory (Hagert, 1985; Hagert and Hansson, 1983, 1984). The process of spatial reasoning thus depends on the total number of mental models required to validate a conclusion. In turn this process depends on the possibility of building an integrated mental representation of the relations among the entities.

1.3.1.3 Relations between Referential Continuity and Referential Determinacy. The above experiments have shown that referential continuity and referential determinacy are critical in constructing mental models and determining the length of a formal derivation. However, the relative importance of the two variables on these processes of spatial reasoning has not yet been investigated. Is referential continuity more important than referential determinacy in determining the difficulty of spatial reasoning?

Table 1 presents four problems which illustrate two levels of the above variables, and the opposing predictions made by Hagert's Formal Rules theory and Johnson-Laird's Mental Models theory concerning

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The symbol <=> denotes a relation of equivalence, and the symbol => denotes a relation of implication.

Table 1. Predictions made by the Mental Models theory and the Formal Rules theory concerning the effects of referential continuity and referential determinacy (as illustrated by problems of type 2 and type 3) on the difficulty of deducing the location of E relative to D.

Referential continuity	Semicontinuous order		Discontinuous order	
Referential determinacy	Problem type 2	Problem type 3	Problem type 2	Problem type 3
Premises	B right of A C left of B D directly below B C directly above E	A right of B C left of B D directly below A C directly above E	E directly below C B directly above D C left of B B right of A	E directly below C A directly above D C left of B A right of B
Question	E left of D?	E left of D?	E left of D?	E left of D?
Number of mental layouts required by the first three premises	1: CAB D 2: ACB D	1: CBA D	1: CB ED	1: CB E 2: A D
Number of mental models for the deduction of E relative to D	1: CAB E D 2: ACB ED	1: CBA E D	1: CAB E D 2: ACB ED	1: CBA E D
Number of steps in the formal derivation of the deduction of E relative to D	1: r(CE) <--> -r(EC) 2: r(EC) & r(CB) => r(EB) 3: r(DB) <--> -r(BD) 4: r(EB) & r(BD) => r(ED)	1: r(CE) <--> -r(EC) 2: r(EC) & r(CB) => r(EB) 3: r(AB) <--> -r(BA) 4: r(EB) & -r(BA) => r(EA) 5: r(DA) <--> -r(AD) 6: r(EA) & r(AD) => r(ED)	1: (EC) & r(CB) => r(EB) 2: r(EB) & r(BD) => r(ED)	1: r(EC) & r(CB) => r(EB) 2: r(AB) <--> -r(BA) 3: r(EB) & -r(BA) => r(EA) 4: r(EA) & r(AD) => r(ED)

their effects. The Formal Rules theory would predict that referential determinacy among premises should prevail over referential continuity in determining the difficulty of spatial reasoning. As illustrated in Table 1, a test of this prediction can be made by presenting problems in the discontinuous order that involves a shorter formal derivation than the semicontinuous order. Thus, the former order should be easier than the latter order. The Mental Models theory makes the opposite prediction as referential continuity is a necessary and prior condition for the integration of a single mental model, and also necessary for the construction of alternative models.

For each level of referential continuity, the Formal Rules theory predicts that problem type 3 should be more difficult than problem type 2, since the former involves a longer formal derivation than the latter. The theory also predicts that problem type 3 in the discontinuous order should be as difficult as problem type 2 in the semicontinuous order since both involve formal derivations of the same length.

The Mental Models theory would make two sets of predictions that depend on the level of referential continuity. In the semicontinuous order, the difficulty of spatial reasoning should depend on the levels of referential determinacy, since the continuity among the premises' entities allows for their continuous integration. Thus, problems of type 2 which yield two mental models should be more difficult than those of type 3 which involve only one mental model.

However, in the discontinuous order, the difficulty of reasoning should be primarily affected by the discontinuity among the premises' entities. As shown in Table 1, the first three premises of problem type 3 generate two independent mental layouts whose relative location is completely indeterminate. These mental layouts can thus be represented in multiple locations in space. In contrast, the first two premises of problem type 2 generate two mental layouts which can be integrated using the third premise. The premises of problem type 3 are thus more likely to interfere with the integration of yet a single mental model than those of type 2.

Problem type 3 (in the discontinuous order) should also be more difficult than problem type 2 (in the semicontinuous order). The first three premises of the latter problem type generate two mental layouts consisting of the same objects, and the relative location of the two mental layouts is irrelevant to the solution of the problem. The representation of two independent mental layouts should thus be more difficult than the representation of two mental layouts having the same entities. Thus, the Mental Models theory predicts that problems which involve two independent mental layouts should be more difficult than problems requiring two related mental layouts or two mental models. The above issues remain open to empirical verification.

1.3.2 Role of Geometrical Content

The Mental Models theory concedes that content factors affect the difficulty of deductive reasoning. The theory proposes two fundamental principles which can elucidate the role of geometrical content on spatial deductive reasoning.

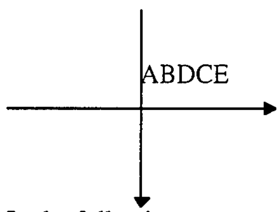
The first principle is that the difficulty of deductive reasoning depends on the extent to which subjects have to flesh out mental models in working memory. The process of fleshing out mental models is a critical aspect of the Mental Models theory, and factors which would induce this process should also elucidate it. A possible way of inducing the process would be to vary the number of dimensions that subjects must deduce between a pair of objects within a mental model. Consider, for instance, a set of premises describing a 2D layout -- A right of B, C left of B, D directly below B, and C directly above E -- and two question types, one concerning the location of E relative to D, and another regarding the location of E relative to B. For the same 2D layout, subjects must locate E relative to D in one dimension, and E relative to B in two dimensions. Thus, to answer each question type, subjects must flesh out their mental models completely. If this process occurs, we would expect the difficulty of spatial reasoning to increase with the number of dimensions that subjects must integrate and inspect within a mental model.

The second principle is that the structure of mental models should be identical to the structure of the relations among entities as they are perceived or conceived. The principle of structural identity implies that mental models should reproduce geometrical relations among entities in a way similar to which mental images reproduce metric relations, such as distances, among a set of entities (Jeannerod, 1994; Kosslyn, Ball, & Reiser, 1978). Kosslyn et al. (1978) have shown that mental images reproduce metric relations as the time required to scan mental images of distances increases with the distances between parts of an imagined object. If mental models actually reproduce geometrical relations and if mental models can themselves be scanned, then the geometrical relations of an argument should affect the time required to construct and scan mental models of such relations.

Structural identity is a fundamental property of mental models, but little is known about this property, and how mental models reproduce the structural relations among entities. One view is to assume that humans structure mental models of geometrical relations relative to spatial reference frames. The use of spatial reference frames implies that different regions of space would be easier to represent and process than others thus affecting the construction of mental models. Logan's (1995) Spatial Reference Frame theory, which we will now consider, may thus provide a complementary account of spatial deductive reasoning.

1.4 Spatial Reference Frame Theory

Logan (1995) argues that humans perceive and represent deictic spatial relations relative to spatial reference frames projected onto space. Deictic spatial relations, such as A is left of B, specify the location of an object (called the located object) relative to another (called the reference object) using projective prepositions such as *left of, above*. A spatial reference frame consists of a set of three coordinate axes (horizontal, vertical, and line of sight) which specify a three-dimensional space. A spatial reference frame has four parameters, an origin, an orientation, a direction, and a scale when a task involves metric relations. In order to process deictic spatial relations, the observer aligns the origin of a spatial reference frame with the reference object (Herskovits, 1986; Logan, 1995). The reference object specifies the orientation and direction of the spatial reference frame. In turn, the spatial reference frame assigns orientation and direction in space. Humans would identify the relative location of an object with respect to the reference object and the spatial reference frame. The latter two are essential in directing spatial attention from one object to another. Take, for example, the following spatial reference frame perceived (or imagined) from a fronto-parallel view perpendicular to the observer's line of sight.



In the following sentence, "B is right of A", the entity A specifies the origin of the spatial reference frame and determines the orientation and the direction from which the entity B will be located. Subsequent sentences such as -- C is right of B, D is directly right of B, C is directly left of E -- specify the location of the entities C, D, and E with respect to B and A since the relations are transitive. Thus, the location of a set of four entities is specified in pairs and relative to the origin A of the spatial reference frame. In the above example, the entities are located from left to right along the horizontal axis, having A as the origin of the axis.

According to Logan, we can infer the use of spatial reference frames when some regions of space are easier to process than others. If euclidian and projective relations affect the difficulty of spatial reasoning, then these effects would indicate that subjects use spatial reference frames in constructing mental models.

Spatial reference frames are flexible means of controlling spatial attention as they can be rotated in different orientations and aligned with any origin. This property provides a basis from which to make predictions concerning the effects of direction while being consistent with the Content-Specific Rules theory (DeSoto et al., 1965). For instance, if a premise set describes an horizontal layout from the directions -- left to right and right to left -- subjects should find it easier to construct mental models from left to right along the horizontal axis rather than in the opposite direction. Similarly, if a premise set describes a vertical layout from the directions -- top to bottom and bottom to top -- subjects would find it easier to construct mental models from top to bottom along the vertical axis rather than in the opposite direction. According to Logan (1995), the latter effect would indicate that subjects rotate their spatial reference frame 90 degrees clockwise from the horizontal axis to align with the vertical axis using the same origin. The ease of constructing mental models from left to right would then be transposed accordingly.

The above effects of projective relations would indicate that spatial reference frames have an orientation and direction from which subjects build mental models. However, if subjects reason about geometrical relations without the use of spatial reference frames, then the equal availability hypothesis (Logan, 1995) should be supported, as there should be no differences in the difficulty of the geometrical relations. Consequently, only the Mental Models theory or the Formal Rules theory should be confirmed. The above issues will now be considered.

2. Objectives

The general aim of this study is to elucidate the effects of the logical form and geometrical content of an argument on the processes that underlie spatial deductive reasoning. We will investigate the effects of these factors by contrasting the opposing predictions made by the following two theories of spatial deductive reasoning: Hagert's Formal Rules theory (Hagert, 1985; Hagert and Hansson, 1983, 1984) which is one of the Formal Rules theories, and Johnson-Laird's Mental Models theory (Johnson-Laird, 1983; Johnson-Laird and Byrne, 1991, 1993). We will also investigate the effects of geometrical content in view of the Content-Specific Rules theory (DeSoto et al., 1968) and the Spatial Reference Frame theory (Logan, 1995). The latter theory will help specify whether humans construct their mental representations of geometrical relations relative to spatial reference frames, and whether these affect the ease of processing geometrical relations according to the Content-Specific Rules theory.

2.1 Role of Logical Form

We will manipulate the effects of *logical form* by varying the continuity and the determinacy of the referents' order in the premise sets. The first variable will be termed "referential continuity" and the second variable "referential determinacy".

2.1.1 Effects of Referential Continuity

Referential continuity will include two levels, a semicontinuous order and a discontinuous order. The semicontinuous order will involve a longer formal derivation than the discontinuous order. Thus, if humans use formal rules of inference to solve the problems, deductions from the discontinuous order should be easier than from the semicontinuous one. However, the discontinuous order will interfere with the continuous integration of the entities into a mental model. Consequently, if humans construct mental models of the premises, the opposite prediction should hold true: the semicontinuous order should be easier than the discontinuous one.

2.1.2 Effects of Referential Determinacy

Referential determinacy will include three levels, called problem type 1, problem type 2, and problem type 3. These problem types will allow comparison of pairs of problems having: (a) formal derivations of the same length but different numbers of mental models (problem type 1 and problem type 2); and (b) formal derivations of different lengths but equal numbers of mental models (problem type 1 and problem type 3). If humans use formal rules of inference, problems of type 3, which involve a longer formal derivation, should be more difficult than those of type 2 and type 1 based on identical but shorter formal derivations. However, if humans build mental models, problems of type 2, which involve two mental models, should be more difficult than those of type 1 and type 3 which yield one mental model.

The semicontinuous order will allow an independent measure of the above effect of referential determinacy; while the discontinuous order will integrate both the effects of referential discontinuity and referential indeterminacy. Thus the latter condition will help elucidate the relative importance of the two variables on the processes of spatial deductive reasoning.

As discussed in the introduction, the first three premises of problem type 3 (in the discontinuous order) will require two independent mental layouts, such as EC and DB, to be held in working memory. Since the two mental layouts are independent, their relative position in space will be completely indeterminate until subjects can use the last premise to integrate them. In contrast, the first three premises of problem type 2 (in the semicontinuous order) will yield two mental layouts, such as ACB and CAB, consisting of the same entities. The relative location of the two mental layouts will be irrelevant to the solution of the problem. The two problem types will involve formal derivations of the same length. If subjects construct mental models, problem type 3 (in the discontinuous order) should be more difficult than problem type 2 (in both orders) as referential continuity will be the critical factor in determining the difficulty of spatial reasoning. Alternatively, if subjects use formal rules of inference, problem type 3 (in the discontinuous order) should be as easy as problem type 2 (in the semicontinuous order). The three problem types should be easier in the discontinuous order than in the semicontinuous one.

2.2 Role of Geometrical Content

We will manipulate the effects of *geometrical content* by varying the euclidean and projective relations among the entities. We will elucidate the effects of *euclidean relations* by varying the number of dimensions described in the premise sets (1D, 2D, and 3D) and question types (1D, 2D, and 3D). The number of dimensions requested in a question type (EB relation and ED relation) will depend on the dimensional condition. We will address the effects of the *projective relations* by varying the orientation (horizontal, vertical) of each layouts and the directions (left/right, above/below) from which they are described along a given orientation. The number of mental models and the number of steps in the formal derivations will be the same across geometrical contents.

The effects of the geometrical content will first address the issue of deductive competence (Evans, 1991) by specifying whether humans are capable of spatial inferences in 3D, and whether they do so using formal rules of inference or mental models. Furthermore, the geometrical content will allow comparison of the opposing predictions made by the Formal Rules theory (Hagert, 1985) and the Mental Models theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1993) of spatial deduction.

If subjects use formal rules of inference to solve the problems, then the geometrical content of the problems should not affect their difficulty since the formal derivations are the same across geometrical contents. Subjects will thus be able of reasoning in 3D as easily as in 1D or 2D.

In contrast, if subjects reason from mental models, the structure of the mental models should be identical to the structure of the relations among the entities. The difficulty of spatial reasoning should thus vary with the nature of the geometrical relations that subjects integrate within a mental model. Moreover, if subjects build mental models relative to spatial reference frames, these should differentially affect the ease of processing geometrical relations. Consequently, the effects of the geometrical content will elucidate the structural properties of mental models, and whether these are constructed relative to spatial reference frames.

The Mental Models theory's principle of structural identity and the Spatial Reference Frame theory (Logan, 1995) allows us to make the following predictions concerning the effects of the euclidian and projective relations.

2.2.1 *Effects of Euclidian Relations*

First, the difficulty of spatial reasoning should increase with the number of dimensions that subjects have to integrate within a mental model during the comprehension stage of deduction. Moreover, since mental models can themselves be scanned during the description stage, then the difficulty of deducing the relative location among two entities should increase with the number dimensions in which the entities are located within a mental model. The number of dimensions described in the problems (premises and questions) should thus increase the information that subjects have to represent explicitly in working memory, and thus increase the difficulty of spatial reasoning. In terms of Logan's theory, the effects of number of dimensions would show that subjects are locating entities within a set of three coordinate axes (horizontal, vertical, and line of sight) which specify the basic properties of a spatial reference frame.

2.2.2 *Effects of Projective Relations*

Subjects may find it easier to construct mental models in one orientation rather than in another, but the effect of orientation remains unclear (Handel, DeSoto, & London, 1968; Jones, 1970). For a given orientation, they may also find it easier to construct mental models from one set of directions rather than from the opposite ones. Given the above two levels of referential continuity, there will be two starting points for each layout, the first entity A and the last entity E. For each starting point, there will be two sets of opposite directions from which the layouts will be described: from right to left and left to right along the horizontal axis, and from bottom to top and top to bottom along the vertical axis. The effect of direction will allow us to determine whether DeSoto et al.'s (1965) directional preference hypothesis generalizes to spatial reasoning. If so, then the effect of the projective relations will help specify the orientation and direction of spatial reference frames in controlling spatial attention during the construction of mental models.

Our last objective is to ascertain the generality of the predictions made by the Formal Rules theory and the Mental Models theory concerning the processes that underlie spatial deductive reasoning. To do so, the experiment will present the problems in a visual mode instead of an auditory one. It has been argued that an auditory mode of presentation would facilitate the construction of mental models while a visual mode of presentation (where premises are written) may interfere with it (Roberts, 1993). Experiments which indicate intermodality interferences, such as those found between reading and spatial visualization (Brooks, 1968), may substantiate this contention. However, this interference is likely to occur when problems measure cognitive strategies rather than basic processes. The fundamental processes that underlie spatial deductive reasoning should be the same regardless of mode of problem presentation.

3. Method

3.1 Subjects

A total of twenty-six subjects (20 males, 6 females; 27 to 47 years old) of various occupational levels (clerk to scientist), military ranks (private to commander), and educational levels (high school to post-graduate degree) completed the experiment. All subjects were paid according to DCIEM guidelines for stress allowance.

3.2 Spatial Deductive Problems

Each spatial deductive problem consisted of four premises and up to three questions. Each premise described the relative location of two objects. Together, these four premises formed a *set* which described the layout of five objects using five one syllable words: bomb, ship, tank, gun, and mine. The subject's task was to determine, from a premise set, whether the relative location of two objects, described in a statement (e.g., Is gun left of mine?), was true or false.

We used two general factors to construct the problems' *premise sets*: *logical form*, which pertained to the order of the objects and *geometrical content* which pertained to the relationships among the objects. The logical form was specified by two variables: referential continuity (2 levels) and referential determinacy (3 levels). The geometrical content was specified by three variables: number of dimension (3 levels), orientation (2 levels), and direction (2 levels). From these variables, we generated a total of 72 premise sets. Each premise set was presented twice once for each of two question types (EB relation and ED relation). The number of questions asked for each *question type* varied between 1 and 3 depending on the number of dimensions described in the premise sets. The experiment comprised a total of 144 problems representing a 2 x 3 x 3 x 2 x 2 x 2 completely crossed factorial design. The number of questions constituted a nested variable.

Appendix A presents the variables and the experimental problems generated from these variables. The appendix also describes the number of mental models, and the length of the formal derivation underlying the solution of each problem. The length of the formal derivation is defined by the minimum number of inferential steps used in logic and are illustrated in the appendix.

3.2.1 Logical Form of the Arguments

3.2.1.1 Referential continuity. There were two levels of referential continuity, a *semicontinuous order* (A-B, C-B, D-B, C-E) in which the third and fourth premise had no referent in common, and a *discontinuous order* (E-C, B-D, C-B, A-B) in which the first and second premise had no referential continuity. We constructed the premise sets of the two orders in pairs using the same relations between the pairs of objects A-B and C-B, but opposite relations between the pairs D-B and B-D on one hand, and between the pairs C-E and E-C on the other hand. This insured that the premises sets in the two orders were topologically equivalent, and yielded, in the discontinuous order a subset of the formal derivation involved by the semicontinuous order (see Appendix A).

3.2.1.2 Referential determinacy. Referential determinacy included three levels called problem type 1, problem type 2, and problem type 3. Problems of type 1 specified the position of the objects unambiguously using the following order between the pairs of objects of each premise: A-B, C-B, D-B, C-E[5] and E-C, B-D, C-B, A-B. For example, the following problem -- 1) A right of B, 2) C left of B, 3) D directly left of B, 4) C directly right of E. Is E left of D? -- yields only one mental model: E C D B A.

Problems of type 2 were the same as those of type 1, except that the order of the objects A-B was inverted to B-A. This inversion caused the location of A to become ambiguous relative to C, given the location of C and B. For example, the following problem -- 1) B right of A; 2) C left of B; 3) D directly left of B; 4) C directly right of E. Is E left of D? -- underlies two mental models: a) E C A D B; b) A E C D B.

According to Hagert's Formal Rules theory (Hagert, 1985; Hagert & Hansson, 1983, 1984), subjects do not require the first premise to deduce the location of E relative to D for the above two types of problems. Since the three remaining premises are identical then both problem types involve the same formal derivation. The Formal Rules theory thus predicts that problems of type 1 and type 2 should be equally difficult.

Problems of type 3 were identical to those of type 1, except that the object D was related to A instead of B. This relation insured that the first premise would be required in the formal derivation of the ED relation. For example, the following problem -- 1) A right of B, 2) C left of B, 3) D directly left of A, 4) C directly right of E. Is E left of D? -- yields one mental model: E C B D A. Yet, it involves a longer formal

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5 During actual experimental runs, all problems used the same set of five objects (bomb, gun, ship, mine, tank) to control for any potential effects due to content other than those investigated in this study. However, we assigned the objects randomly to the entities A, B, C, D, and E, thus forming for each subject different sets of lexical tokens.

derivation than problems of type 1 and type 2 (see Appendix A). The formal derivations required to deduce the EB relation were identical for all three problems types, but were shorter than for the ED relation.

3.2.2 Geometrical Content of the Arguments

3.2.2.1 Number of dimensions. We varied the number of dimensions described in the premise sets according to three dimensional conditions: 1D, 2D, and 3D. For the 1D condition, the premise sets described 1D layouts along either the horizontal or the vertical axis. For the 2D condition, the premise sets described 2D layouts along the horizontal and vertical axes; and for the 3D condition, the premise sets described 3D layouts along the horizontal, vertical, and line of sight axes.

The number of *dimensions* pertaining to the relative location of the pair of objects ED and EB depended on the dimensional condition. For the 1D problems, subjects were required to deduce the relative location of ED and EB in one dimension (e.g., Is E left of D?) either along the horizontal or the vertical axis. Thus, only one question was asked. For the 2D problems, subjects were to deduce the relative location of ED in one dimension (as for the 1D problems), and that of EB in two dimensions. For the 3D problems, subjects were to deduce the relative location of ED in three dimensions, and that of EB in two dimensions (as for the 2D problems).

The number of mental models and the number of steps in the formal derivation of each question type were the same for all three dimensional conditions (see Appendix A). However, the formal derivation of ED was longer than that of EB regardless of the number of dimensions in which the pair of objects were located.

3.2.2.2 Orientation. The premise sets described the layouts according to an horizontal axis and a vertical axis. For the 1D condition, the premise sets specified the location of all five objects along either one or the other of these axes. For the 2D and 3D conditions, the relative location of the objects A, B, and C, which formed the main axis of the layouts, determined their orientation. The vertical layouts corresponded to a 90 degree clockwise rotation of the horizontal layouts. The layouts were topologically equivalent.

3.2.2.3 Direction. The premise sets described the layouts starting with the object A (for the semicontinuous order) or the object E (for the discontinuous order) as the point of departure from which the layouts were described in different directions. These points of departure did not constitute an independent variable as they resulted from the use of two levels of referential continuity.

As indicated, subjects were to assume that the five entities were located in a fronto-parallel view perpendicular to their line of sight ("The objects are displayed directly in front of you"). Starting from the object A, the horizontal layouts were described from right to left (H1) and left to right (H2) along the horizontal axis; and the vertical layouts were described from bottom to top (V1) and top to bottom (V2) along the vertical axis. Starting from the object E, the premise sets described the layouts from the opposite direction from which they were described from the object A. For the 1D condition, these directions applied unequivocally to all five objects which were located either along the horizontal axis or the vertical axis. For the 2D and 3D conditions, the relative location of the objects A, B, and C, which formed the main axis of the layout specified the direction from which they were described along the horizontal and vertical axis.

3.2.3 Controls

To insure that each problem generated one valid conclusion, we applied three controls. The first involved instructing the subjects to assume that the objects were located directly in front of them, i.e., in a fronto-parallel plane. The second concerned the use of the term "directly" in the premises relating the pairs of objects D-B, D-A, and C-E (e.g., C directly below E). The term implied collinearity and that no object or space intervened between the two in question.

However, the term "directly" was not used in the premises relating the pairs of objects A-B (or B-A) and C-B. The use of this term would have compromised problem type 2 in which the location of C must be indeterminate with respect to A, a requirement for testing the principle predictions of the Formal Rules theory and the Mental Models theory.

Finally, to insure that the relative location of the objects A, B, C was indeed indeterminate while positioned on a single axis, subjects were told to assume that these objects were collinear on given axis. Using the following problem of type 2 -- 1) B right of A; 2) C left of B; 3) D directly left of B; 4) C directly right of E. Is E left of D? -- and given that the objects A, B, and C are collinear horizontally, the premises are consistent with two mental models: a) E C A D B; b) A E C D B.

It is important to recognize that without the above controls, the problems yield an infinite number of formal derivations and mental models. Therefore, in order to carefully test the predictions of the Formal Rules theory and the Mental Models theory, these controls are essential.

3.3 Procedure

Subjects participated individually in sound-attenuated rooms each equipped with a 386 PC for problem generation and a two-button mouse for responses. We used a closed-circuit television to monitor the experimental sessions.

The participants received six practice problems followed by the experimental problems. All problems were presented in a visual sequential mode where each premise and each question was presented individually according to predefined temporal parameters. These were determined from a pilot study in which we manipulated the temporal parameters of stimuli presentation to insure a success rate between 60% and 90%. We used the following temporal parameters:

- display time of each premise: 4 seconds
- interpremise time: 5 seconds
- interquestion time: 400 ms
- interproblem time: 10 seconds.

Each premise remained visible on the computer screen for 4 seconds. A blank screen of 5 seconds intervened between each premise, and between the last premise and the first (or only) question associated with a question type. Subjects were then shown up to three consecutive questions that required them to validate a stated location between two objects (e.g., is E left of D?). The truth or falsity of the answer was varied randomly for each subject, problem, and question.

Each question remained displayed on the computer screen until the subject responded by selecting the left mouse button (yes: the relative location is true), the right mouse button (no: the relative location is false), or any key on the keyboard if they did not know the answer. The latter were treated as incorrect.

We instructed the subjects to solve each problem as quickly and as accurately as possible. Their response times were measured to the nearest millisecond from the onset of a question on the computer screen. A blank screen of 400 milliseconds separated the subject's answer to one question and the presentation of the next question (that is, in instances where a question type involved more than one question). A blank interval of 10 seconds separated each problem.

We presented the entire set of 144 problems in eight sessions lasting approximately 15-20 minutes each. These sessions occurred on two different days, each involving four sessions. A session consisted of 18 problems, half of which involved the ED relation, and the other half, the EB relation. We presented the four premises of a set in a predefined order (see Appendix A). However, each subject received a question type and related questions in a different random order to free the data from any potential practice and/or sequence effects.

After completing half (72 problems) of the entire set of problems and also after the entire set (144 problems), the experimenter asked the subject to explain how s/he organized the information described in the premises, and how s/he solved the problems. We tape recorded each subject's explanations for further analysis concerning the nature of the representation and processes underlying spatial deductive reasoning (Evans, 1991; Evans, Newstead, and Byrne, 1993; Roberts, 1993).

4. Analyses

Univariate analyses of variance for repeated measures were performed on the response times (RTs) obtained for the correct responses and on the percentages of correct responses (CRs)[6]. RTs were normally distributed and varied between 2 seconds and 85 seconds. Consequently, the analyses of variance were carried out directly on the RTs.

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6 To stabilize the variance of each effect, we applied an arcsine transformation (Federer, 1963) on the percentages of CRs obtained for each subject and each experimental condition. When applied to the arcsine of the percentages of CRs, the analyses of variance lead to the same results as those obtained before this transformation. Therefore, we will present only the untransformed results.

In the experimental design, the levels of each independent variable were completely crossed with regards to the first question (see Appendix A). The design of the main analysis of variance thus involved 6 variables: referential continuity (2) x referential determinacy (3) x dimensional condition (3) x orientation (2) x direction (2) x question type (2).

Because the 2D questions were nested within dimensional conditions 2D and 3D, and the 3D questions were nested only within dimensional condition 3D, we assessed the effects of these questions by carrying out two additional analyses of variance. One analysis was performed on dimensional conditions 2D and 3D for the EB relation which allowed comparison of the 2D questions. The other analysis was performed on dimensional condition 3D for the ED relation which allowed comparison of the 3D questions.

We performed Geiser-Greenhouse epsilon corrections to adjust the degrees of freedom of each effect. Contrasts between pairs of means were calculated for the levels of the significant effects set at the probability of 0.05. However, as we will note, all but a few of the significant effects were reliable at $p < 0.01$.

5. Results and Discussion

The results are presented in two sections. Section 5.1 will elucidate the role of logical form as measured by the effects of referential continuity and referential determinacy. Section 5.2 will assess the effects of the euclidean and projective relations.

5.1 Role of Logical Form

5.1.1 Effects of Referential Continuity

The Formal Rules theory (Hagert, 1985; Hagert & Hansson, 1983, 1984) predicts that problems presented in a semicontinuous order should be more difficult than those presented in a discontinuous order since the former order involves a longer formal derivation than the latter. The Mental Models theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1993) makes the opposite prediction based on the number of mental layouts that subjects must hold in working memory before their integration into a mental model.

As predicted by the Mental Models theory, the semicontinuous order was significantly easier than the discontinuous order in terms of RTs [$F(1, 25) = 12.71, p < 0.01$] and CRs [$F(1, 25) = 14.26, p < 0.01$] (see Table 1). There was no significant interaction between referential continuity and question type (EB relation and ED relation) [RTs: $F(1, 25) = 1.56$; CRs: $F(1, 25) = 5.22$]. These results also refute the Formal Rules theory's predictions since the EB relation, which involves a shorter formal derivation, should have been easier to deduce than the ED relation (see Appendix A).

The effect of referential continuity interacted significantly with that of referential determinacy [RTs: $F(2, 50) = 5.59, p < 0.01$; CRs: $F(2, 50) = 9.73, p < 0.01$]. This interaction is important since in the semicontinuous order the effect of referential indeterminacy is manipulated independently from that of

Table 2. Mean Response Times and Percentages of Correct Responses obtained for the Arguments Structured according to Logical Form and Geometrical Content

Independent variables	Mean RTs	Percentages of CRs
Logical Form		
Referential continuity		
Semicontinuous order	7.3	.81
Discontinuous order	8.0	.72
Referential determinacy		
Problem type 1	7.4	.77
Problem type 2	7.9	.77
Problem type 3	7.7	.76
Geometrical content		
Dimensional condition		
1D	6.8	.81
2D	7.9	.76
3D	8.3	.73
Orientation		
Horizontal	7.6	.77
Vertical	7.8	.76
Direction		
Right to left	7.1	.81
Left to right	8.1	.72
Top to bottom	7.2	.81
Bottom to top	8.2	.72

referential discontinuity; while in the discontinuous order, the two factors are integrated. The latter order will thus help specify the relative importance of the two factors.

5.1.1.1 Effects of Semicontinuous Order. As shown in Figure 1, in the semicontinuous order, problems of type 2 based on two mental models were significantly more difficult than problems of type 1 [RTs: $F\ (1, 25) = 7.06$, $p < 0.01$; CRs: $F\ (1, 25) = 2.03$, $p > 0.01$] and type 3 [RTs: $F\ (1, 25) = 9.53$, $p < 0.01$; CRs: $F\ (1, 25) = 7.35$, $p < 0.01$] respectively. The difficulty of the latter two did not differ reliably [RTs: $F\ (1, 25) = .18$; CRs: $F\ (1, 25) = 1.66$] despite the fact that problems of type 3 required a longer formal derivation than those of type 1. There was no significant interaction between the effects of referential continuity, referential determinacy, and dimensional condition [RTs: $F\ (4, 100) = 1.15$; CRs: $F\ (4, 100) = .25$]. These results thus confirm the Mental Models theory's principle predictions which now generalize to spatial reasoning in all three dimensions. The results also corroborate those obtained by Byrne and Johnson-Laird (1989) who presented 2D problems in a semicontinuous order. In both studies, problems based on two

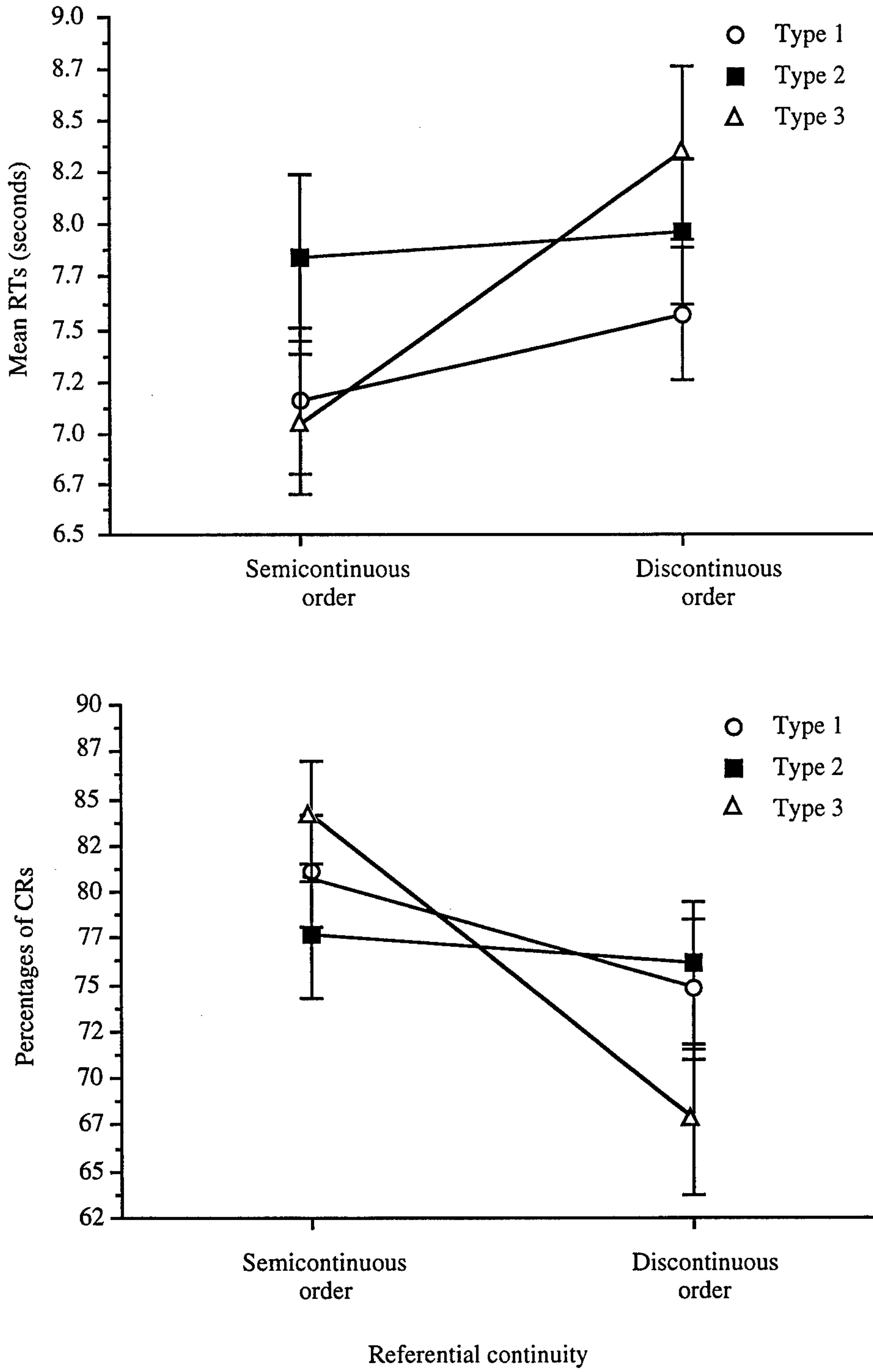


Figure 1. Mean response times and percentages of correct responses obtained for each problem type and each level of referential continuity

mental models (type 2) were reliably more difficult than those based on one mental model (type 3 and type 1) despite the fact that problems of type 2 involved a shorter formal derivation than those of type 3.

5.1.1.2 Effects of Discontinuous Orders. In the discontinuous order, problems of type 3 were more difficult than those of type 1 [RTs: $F\ (1, 25) = 8.91, p < 0.01$; CRs: $F\ (1, 25) = 10.87, p < 0.01$] and those of type 2 in terms of accuracy [RTs: $F\ (1, 25) = 2.12, p > 0.01$; CRs: $F\ (1, 25) = 12.43, p < 0.01$]. Problems of type 1 and type 2 did not differ reliably (RTs: $F\ (1, 25) = 2.34$). Problems of type 3 in the discontinuous order were also systematically more difficult than those of type 2 in the semicontinuous order [RTs: $F\ (1, 25) = 3.71, p < 0.05$; CRs: $F\ (1, 25) = 17.67, p < 0.01$] although both problem types required formal derivation of the same length. The above changes in the patterns of difficulty are related to the fact that problems of type 3 and of type 1 were systematically more difficult in the discontinuous order than in the semicontinuous one despite the fact that the former order required a shorter formal derivation than the latter order (type 3 [RTs: $F\ (1, 25) = 25.14, p < 0.01$; CRs: $F\ (1, 25) = 47.81, p < 0.01$]; and type 1 [CRs: $F\ (1, 25) = 12.40, p < 0.01$; RTs: $F\ (1, 25) = 2.56, p > 0.01$]. Problems of type 2 were more difficult in the discontinuous order than in the semicontinuous one but the differences were not reliable [RTs: $F\ (1, 25) = .22$; CRs: $F\ (1, 25) = .46$].

Since all problem types should have been easier in the discontinuous order, the above results thus contradict the effects predicted by the Formal Rules theory. Instead, they show that referential discontinuity has more impact on the difficulty of spatial reasoning than referential indeterminacy, thus corroborating the Mental Models theory.

In the discontinuous order, the first three premises of problems of type 3 required subjects to hold two independent mental layouts in working memory for 27 seconds, such as:

CB   A

E   D.

Since the two mental layouts were independent, their relative position in space was completely indeterminate until subjects could use the last premise to integrate them into a mental model. In the discontinuous order, the first two premises of problems type 2 and type 1 generated two independent mental layouts that subjects had to hold for 18 seconds in working memory. However, the third premise yielded one mental layout. Problems of type 2 were as difficult in the discontinuous order as in the semicontinuous one. These results thus suggest that the third premise of the four was critical in determining the ease with which subjects were able to construct a mental model. Since problem type 3, in the discontinuous order, was also systematically more difficult than problem type 2 in the semicontinuous order, this result indicates that the representation of two independent mental layouts (required by the first three premises of problem type 3) is more difficult than the representation of two mental layouts consisting of the same objects (required by the first three premises of problem type 2), and that of two mental models (required by all premises of problem type 2). Thus, referential continuity is the critical factor in determining the difficulty of spatial reasoning. When discontinuity among referents occurs throughout the first three premises, subjects take more time and are less accurate in constructing even a single mental model.

Overall, the effect of referential continuity, as measured in terms of RTs, did not interact significantly with the geometrical content of the premises, that is with either the dimensional conditions [$F\ (2, 50) = .59$], orientations [$F\ (1, 25) = .59$], or directions [$F\ (1, 25) = 2.64$]. In terms of CRs, there were also no significant interaction (dimensional conditions [$F\ (2, 50) = 1.35$], orientations [$F\ (1, 25) = .51$]) with the single exception of a significant interaction between referential continuity and direction [$F\ (1, 25) = 9.32$, $p < 0.01$]. This interaction will be considered in section 5.2.

The effects of referential continuity corroborates the Mental Models theory's predictions, since the continuity between the premises' entities significantly facilitated the ease with which subjects integrated these entities. In contrast, referential discontinuity increased the difficulty of this spatial integration as subjects were required to hold mental layouts independently in working memory. The effects of referential continuity are also consistent with those obtained in previous studies on the representation of nonspatial relations (Foos, Smith, Sabol, & Mynatt, 1976; Mynatt & Smith, 1977) and that of 2D spatial relations (Ehrlich & Johnson-Laird, 1982). However, the foregoing results generalize the role of referential continuity to the process of spatial reasoning in all three dimensions. They also show that in the discontinuous order, the representation of two independent mental layouts is more difficult than the representation of two related mental layouts or two mental models. Thus, referential discontinuity takes precedence over referential indeterminacy in determining the difficulty of spatial reasoning.

5.1.2 Effects of Referential Determinacy

The Formal Rules theory makes two sets of predictions for the effects of referential determinacy which depend on the question type (ED relation and EB relation). For the ED relation, problem type 1 should be as easy as problem type 2, and both should be easier than problem type 3 which involves a longer formal derivation than the former two. For the EB relation, all three problem types should be equally difficult since they involve identical formal derivations. In contrast, the Mental Models theory predicts that problem type 1 should be as easy as problem type 3, and both should be easier than problem type 2 since the former two yield one mental model while the latter yields two mental models.

Referential determinacy had a significant main effect on the RTs [$F(2, 50) = 10.19$, $p < 0.01$] but not on the CRs [$F(2, 50) = .19$]. Referential determinacy did not interact significantly with the question types either in terms of RTs [$F(2, 50) = .11$] or CRs [$F(2, 50) = .18$]. These results refute the Formal Rules theory's prediction since the difficulty of the problem types should have varied with the question types.

As shown in Table 1, problems of type 2, based on two mental models, took more time to solve than those of type 1 [$F(1, 25) = 20.06$, $p < 0.01$]. The former were also more difficult than problems of type 3 but the differences were not significant [$F(1, 25) = 3.04$]. As discussed in the preceding section, in the semicontinuous order, problem of type 2 were significantly more difficult than those of type 3 [RTs: $p < 0.01$; CRs: $p < 0.01$], while in the discontinuous order problem type 3 were more difficult than problem type 2 (CRs: $p < 0.01$). The interaction between the effects of referential continuity and referential determinacy can thus account for the lack of significant differences between problem type 2 and problem type 3.

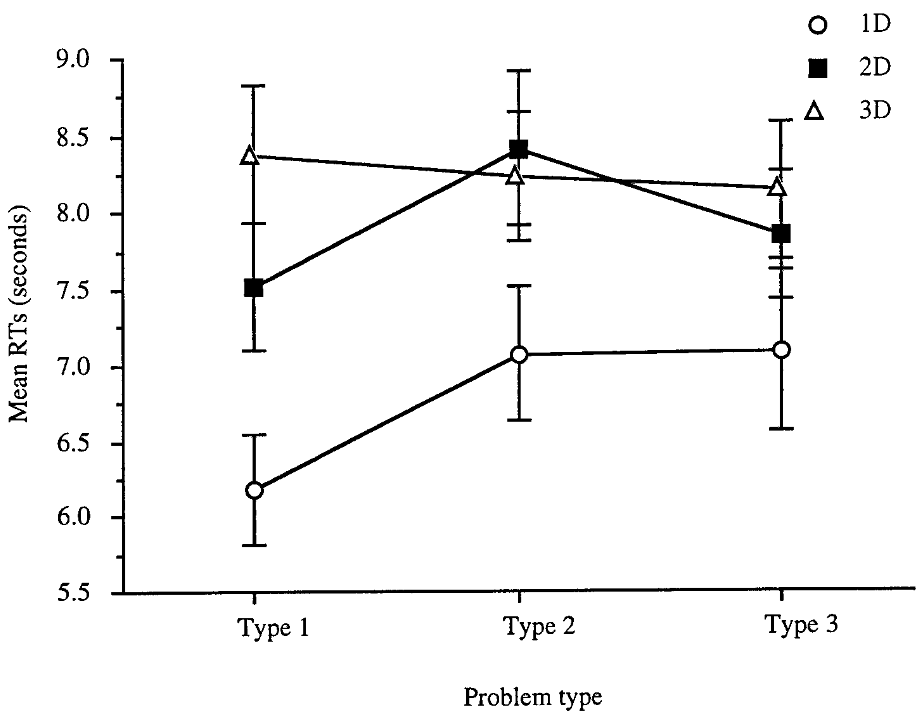


Figure 2. Mean response times obtained for each problem type and each dimensional condition

The same interaction can also partly account for the fact that the effect of referential determinacy interacted significantly with the dimensional conditions in terms of RTs [RTs: $F(4, 100) = 3.96$, $p < 0.01$; CRs: $F(4, 100) = .66$]. As shown in Figure 2, for the 1D condition, problems of type 1 (based on one mental model) were significantly easier than problems of type 2 (based on two mental models) [$F(1, 25) = 12.07$, $p < 0.01$] and type 3 [$F(1, 25) = 12.60$, $p < 0.01$]. Problems of type 2 and type 3 did not differ reliably [$F(1, 25) = .01$]. For the 3D condition, the three problem types did not differ significantly from one another [(type 1 vs. type 2: $F(1, 25) = .31$), (type 1 vs. type 3: $F(1, 25) = .84$), (type 2 vs. type 3: $F(1, 25) = .13$)]. However, for the 2D condition, problems of type 2 required significantly more time to solve than problems of type 3 [for both question types: $F(1, 25) = 4.89$, $p < 0.02$; and for the ED relation: $F(1, 25) = 7.55$, $p < 0.01$] and type 1 [$F(1, 25) = 12.17$, $p < 0.01$] respectively. The latter two problem types, which were based on one mental model, did not differ significantly [$F(1, 25) = 1.64$] despite the fact that problems of type 3 involved a longer formal derivation than those of type 1. For the 2D condition, the RTs obtained for the three problem types thus confirm each of the Mental Models theory's predictions.

Since the 3D condition resulted in the longest RTs, the lack of differences between the three problem types may reflect a ceiling effect due to the difficulty of the condition and to the temporal constraints under which we presented the problems. The temporal parameters may not have provided subjects enough time to construct alternative mental models of the premises. If subjects could control the display times of each premise, the above ceiling effect might be removed.

It remains the case however, that when the effects of referential determinacy are manipulated independently from those of referential continuity, the Mental Models theory's predictions are confirmed and generalize to spatial reasoning in all three dimensions. Thus, for the semicontinuous order, problems based on two mental models (problem type 2) were reliably more difficult than problems based on one mental model (problems type 1 and type 3) although the former required a shorter formal derivation than problem type 3. This above assertion is supported by the fact, noted in section 5.1.1, that there were no significant interactions between the effects of referential continuity and dimensional condition, or the latter two factors and referential continuity.

5.2 Role of Geometrical Content

The geometrical content pertained to the euclidian relations among the entities, that is, the number of dimensions described in the premise sets (1D, 2D, and 3D) and question types (1D, 2D, and 3D); and the projective relations among the entities, that is, the orientation of the layouts, and the directions in which the layouts were described along a given orientation.

5.2.1 Effects of Euclidian Relations

The Formal Rules theory predicts that the number of dimensions stated in the problems' premise sets should not affect the difficulty of the problems as they involve identical formal derivations. The theory also predicts that the ED relation should be more difficult than the EB relation since the former involves a longer formal derivation than the latter. This difference in difficulty between the two question types should hold true irrespective of the number of dimensions involved in the question types.

The Mental Models theory makes two alternative predictions regarding the effects of number of dimensions. First, if the difficulty of spatial reasoning depends essentially on the number of mental models consistent with a problem, then there should be no differences between the difficulty of the dimensional conditions or that of the question types since they involve the same number of mental models. However, if mental models reproduce euclidian relations, then the difficulty of spatial deductions should increase with the number of dimensions that subjects must integrate and inspect within a mental model. The effect of the euclidian relations should thus indicate that subjects are using a spatial reference frame to construct mental models.

5.2.1.1 Effects of Dimensional Condition. The ease with which subjects solved the problems decreased with the increasing number of dimensions described in the problems' premise sets (see Table 1). The effects of the dimensional conditions were also highly reliable for both the RTs [$F(2, 50) = 29.86$, $p < 0.01$] and the CRs [$F(2, 50) = 11.63$, $p < 0.01$].

Contrasts indicate that the mean RTs increased significantly from 1D to 2D [$F(1, 25) = 32.90$, $p < 0.01$] for both question types pooled, and from 2D to 3D but for the ED relation only [$F(1, 25) = 4.55$, $p < 0.03$]. Likewise, and for both question types, the percentages of CRs decreased significantly from 1D to 2D [$F(1, 25) = 8.31$, $p < 0.01$], and from 2D to 3D [$F(1, 25) = 4.57$, $p < 0.03$]. These results refute the Formal Rules theory prediction's since all dimensional conditions involved formal derivations of the same length. Instead, they indicate that subjects build mental models which reproduce the euclidian relations among the entities according to a set of three coordinate axes which form the basis of a spatial reference frame.

However, the results differ from those obtained by Byrne & Johnson-Laird (1989). In that experiment, subjects were just as accurate in solving 2D problems as the 1D ones. The differences in the results may be due to the differences in the experimental design, and in particular the temporal constraints under which we presented the problems to the subjects of this experiment. In Byrne & Johnson-Laird's experiment, the premises' display times were probably longer than those used in the present experiment thus allowing subjects to construct 2D mental models as easily as 1D ones.

5.2.1.2 Effects of Number of Dimensions in Question Type. As indicated, the number of dimensions requested in a question type constituted a nested variable. Consequently, separate analyses of variances were performed to assess the variable's effect for both within and between the 1D, 2D and 3D problems. For the 1D problems, subjects were to deduce the relative location of ED and that of EB in one dimension. The analysis of variance performed on the 1D problems indicate that the two question types were of similar difficulty in terms of RTs [$F(1, 25) = .002$] and CRs [$F(1, 25) = 3.23$] (see Figure 3). For the 2D problems, subjects were to deduce the relative location of ED in one dimension (as for the 1D problems), and the relative location of EB in two dimensions (as for the 3D problems). The relative location of ED in one dimension was as easy to deduce for the 2D problems (CRs: 78%) as for the 1D problems (CRs: 79%) but it took less time [$F(1, 25) = 22.35$, $p < 0.01$] for the 1D problems (see Figure 3). For the 1D problems, the relative location of EB in one dimension was significantly easier to deduce than that of EB for either the 2D problems [CRs: $F(1, 25) = 15.75$, $p < 0.01$; RTs: $F(1, 25) = 28.32$, $p < 0.01$] or the 3D problems [CRs: $F(1, 25) = 9.48$, $p < 0.01$; RTs: $F(1, 25) = 37.66$, $p < 0.01$].

For the 2D problems, the relative location of ED in one dimension was easier to deduce than that of EB in two dimensions [CRs: $F(1, 25) = 5.05$, $p < 0.02$; RTs: $F(1, 25) = 100.26$, $p < 0.01$]. The Formal Rules theory would have predicted the inverse order of difficulty as the EB relation involved a shorter formal derivation than the ED relation (see Appendix A).

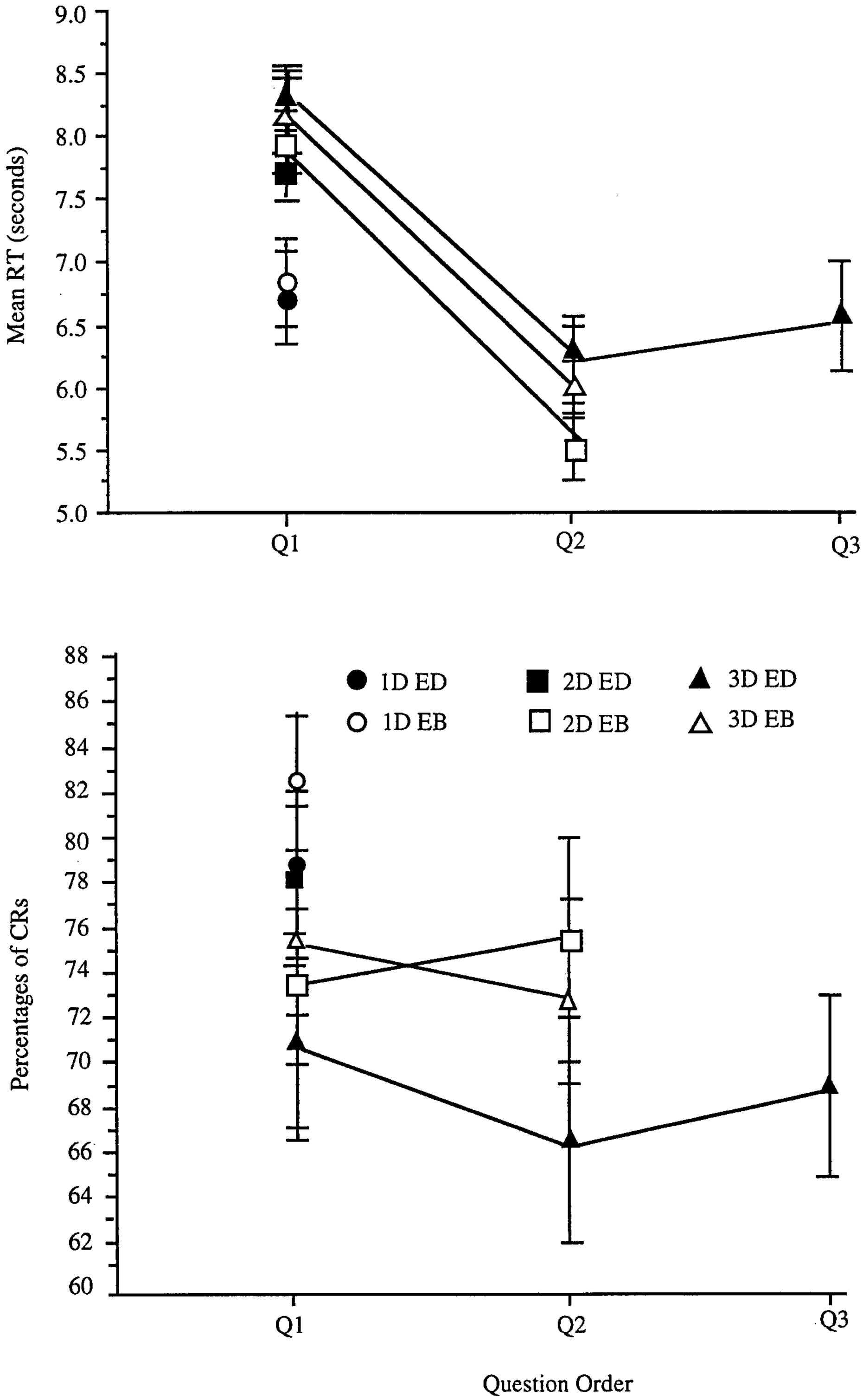


Figure 3. Mean response times and percentages of correct responses obtained for each dimensional condition, each question type (ED relation and EB relation) and each associated question

For the 3D problems, subjects were to deduce the relative location of ED in three dimensions, and that of EB in two dimensions (as for the 2D problems). The relative location of ED in three dimensions was more difficult to deduce than that of EB in two dimensions [CRs: $F(1, 25) = 5.49, p < 0.02$; RTs: $F(1, 25) = 1.83, p > 0.01$]. Also, the relative location of EB in two dimensions was as easy to deduce for the 3D problems as for the 2D ones [RTs: $F(1, 25) = 3.19$; CRs: $F(1, 25) = .98$].

The above results indicate that when subjects had the same number of dimensions to locate between the pair of objects ED or EB, the difficulty of their deductions was the same despite the fact that the ED relation involved a longer formal derivation than the EB relation. Likewise, when subjects had increasing numbers of dimensions to locate between the pair of objects, the difficulty of their deductions increased accordingly irrespective of the length of the formal derivation underlying the deduction of the relative location of ED or EB.

These results thus refute the Formal Rules theory's predictions. In fact, for the 2D problems, the effect of question type was quite the opposite of the theory's predictions. The results are thus consistent with and extend the Mental Models theory's predictions. They show that mental models reproduce the euclidian relations among the entities as they are conceived (Johnson-Laird and Byrne, 1993) in a way similar to which visual images reproduce metric relations among entities (Jeannerod, 1994; Kosslyn et al., 1978). The results thus indicate the presence of a spatial reference frame during the construction of mental models since the three coordinate axes differentially affected the difficulty of spatial reasoning.

5.2.2 Effects of Projective Relations

As for the euclidian relations, the Formal Rules theory predicts that the projective relations (orientation and direction) should not affect the difficulty of the problems since they involve, within problem types, identical formal derivations. Also, irrespective of the directions, the ED relation should be more difficult to deduce than the EB relation. In contrast, the Mental Models theory concedes that the projective relations may affect the difficulty of spatial reasoning for the similar reasons as those invoked for the euclidian relations. Thus, if mental models reproduce the structure of the projective relations, these should differentially affect the ease of constructing mental models. DeSoto et al.'s (1965) principle of directional preference will be used to predict the effects of direction, although those of orientation remain unclear. The effects of the projective relations will help elucidate two basic parameters of the spatial reference frames, namely their orientation and direction (Logan, 1995). These four theories will thus provide a theoretical basis from which to analyze the effects of orientation and direction.

5.2.2.1. Effects of Orientation. The premise sets described the layouts according to an horizontal axis and a vertical axis. For the 1D condition, the premise sets specified the location of all five objects along either one or the other of these axes. For the 2D and 3D conditions, the relative location of the objects A, B, and C, which formed the main axis of the layouts, determined their orientation.

Overall, the effect of orientation was not significant either in terms of RTs [$F(1, 25) = .55$] or CRs [$F(1, 25) = .08$]. The effect of orientation did not interact significantly with the variables pertaining to the logical form of the premises (referential continuity [RTs: $F(1, 25) = .59$; CRs: $F(1, 25) = .51$]; referential determinacy [RTs: $F(2, 50) = .61$; CRs: $F(2, 50) = .21$]), their geometrical content (dimensional condition [RTs: $F(2, 50) = 2.50$; CRs: $F(2, 50) = .89$]; direction [RTs: $F(1, 25) = .003$; CRs: $F(1, 25) = .03$]), or the question types [RTs: $F(1, 25) = .01$; CRs: $F(1, 25) = .39$]. There were no three-way significant interactions between orientation and any other two variables ($p > 0.01$).

However, a separate analysis of variance performed on the 3D condition indicates that subjects drew spatial inferences more rapidly from the horizontal layouts (mean RTs = 6.80 sec.) than from the vertical ones (mean RTs = 7.25 sec.). The differences were also reliable in terms of RTs [RTs: $F(1, 25) = 5.69$, $p < 0.02$; CRs: $F(1, 25) = .21$]. This result suggests that in building 3D mental models, the horizontal axis facilitates the visualization of objects along the line of sight axis, possibly because humans generally view objects in depth relative to the horizon. But for the 1D and 2D conditions, subjects drew spatial inferences just as easily from the horizontal layouts as from the vertical ones. Given Logan's (1995) equal availability hypothesis, these results suggest that the horizontal and vertical axes are equally easy to process while the line of sight axis is easier to process relative to the horizontal axis than to the vertical axis.

5.2.2.2 Effects of Direction. The premise sets described the layouts starting with the object A (for the semicontinuous order) or the object E (for the discontinuous order) as the point of departure from which the layouts were described in different directions. These points of departure did not constitute an independent variable as they resulted from the use of two levels of referential continuity.

Starting from the object A, the horizontal layouts were described from right to left and left to right along the horizontal axis; and the vertical layouts were described from bottom to top and top to bottom along the vertical axis. Starting with the object E, the premise sets described the layouts from the opposite direction from which they were described from the object A.

As shown in Table 1, direction had a main effect on both the RTs [$F(1, 25) = 126.10$, $p < 0.01$] and the CRs [$F(1, 25) = 38.11$, $p < 0.01$] thus indicating that subjects were constructing mental models. Overall, there were no significant interactions between the effects of direction and either those of referential determinacy [RTs: $F(2, 50) = 1.15$; CRs: $F(2, 50) = 1.06$], those of referential continuity [RTs: $F(1, 25) = 2.64$; CRs: $F(1, 25) = 9.32$, $p < 0.01$], or those of orientation [RTs: $F(1, 25) = 3.64$; CRs: $F(1, 25) = .001$]. Also, the effects of direction did not interact significantly with those of referential continuity and orientation [RTs: $F(1, 25) = 3.64$; CRs: $F(1, 25) = .001$].

As illustrated in Figure 4, in the semicontinuous order, mental models were significantly easier to construct from right to left along the horizontal axis, and from bottom to top along the vertical axis rather than from the respective opposite directions [RTs: $F(1, 25) = 26.56$, $p < 0.01$; CRs: $F(1, 25) = 7.37$, $p < 0.01$]. Conversely, in the discontinuous order, mental models were significantly easier to construct from left to right along the horizontal axis, and from top to bottom along the vertical axis rather than from the respective opposite directions [RTs: $F(1, 25) = 55.54$, $p < 0.01$; CRs: $F(1, 25) = 49.44$, $p < 0.01$]. As indicated above, for the CRs the effect of direction interacted significantly with that of referential continuity

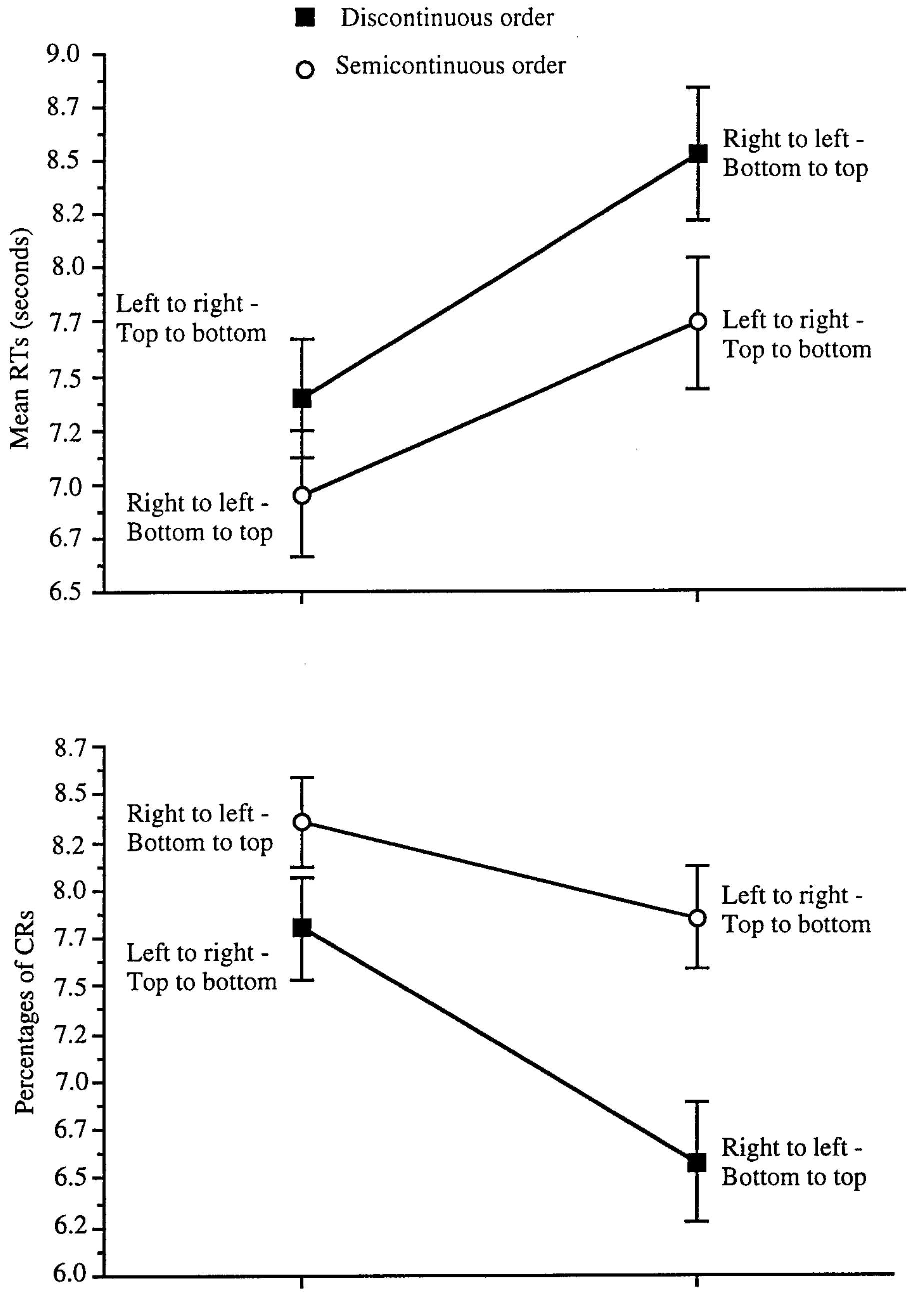


Figure 4. Mean response times and percentages of correct responses obtained for each level of referential continuity and each direction from which the layouts were described

($p < 0.01$). The interaction is due to the fact that mental models were systematically easier to construct in the semicontinuous order starting from left to right or top to bottom rather than in the discontinuous order starting from right to left or bottom to top [CRs: $F(1, 25) = 52.97$, $p < 0.01$]. However, when subjects had to construct mental models from left to right, or top to bottom, they were just as accurate in building these mental models in the semicontinuous order as in the discontinuous order [CRs: $F(1, 25) = .06$], but it took them more time in the latter order [RTs: $F(1, 25) = 5.19, p < 0.01$]. Overall, these results confirm the Content Specific Rules theory's predictions (DeSoto et al., 1965). The effects of direction depend however on whether the layouts were described in a semicontinuous order or in a discontinuous order.

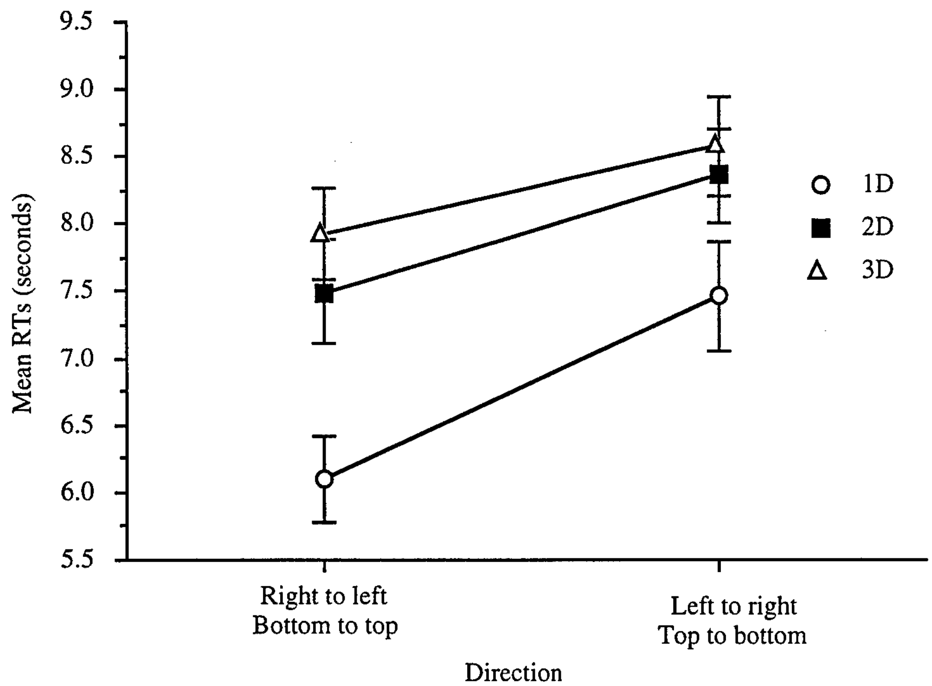


Figure 5. Mean response times obtained for each dimensional condition and for each direction from which the layouts were described

As illustrated in Figure 5, the time required to construct mental models from the different directions increased significantly with the dimensional conditions [RTs: $F(2, 50) = 4.27, p < 0.01$; CRs: $F(2, 50) = 1.71$]. Contrasts indicate that, for the semicontinuous order, the difficulty of constructing mental models from right to left, and bottom to top increased reliably from 1D to 2D [RTs: $F(1, 25) = 61.62, p < 0.01$], and from 2D to 3D [RTs: $F(1, 25) = 5.76, p < 0.02$]. For the discontinuous order, the difficulty of constructing mental models from left to right (and top to bottom) also increased systematically from 1D to 2D [RTs: $F(1, 25) = 25.09, p < 0.01$], and from 2D to 3D but the latter differences were not significant [RTs: $F(1, 25) = 1.50$]. These results are consistent with the effects of dimensional condition.

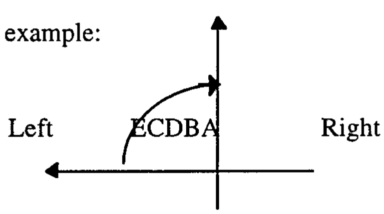
One might expect an interaction between the three factors of direction, referential continuity, and dimensional condition since only the 1D condition represented the horizontal and vertical axes uniformly. However, the interaction between the three factors was not significant [RTs: $F(2, 50) = 1.24$; CRs: F (2,

50) = .05]. The above effects of direction were thus consistent and generalized across all three dimensional conditions.

The effects of direction contribute, in four ways, to the theories of spatial deductive reasoning. First, they refute the Formal Rules theory's (Hagert, 1985) predictions since problems based on different directions involved identical formal derivations, and since the two question types were equally difficult. The results suggest instead that subjects construct mental models (Johnson-Laird & Byrne, 1991) which reproduce projective relations according to systematic patterns of *directional preferences* (DeSoto et al., 1965). Hence, the results provide additional evidence that subjects build mental models relative to spatial reference frames (Logan, 1995).

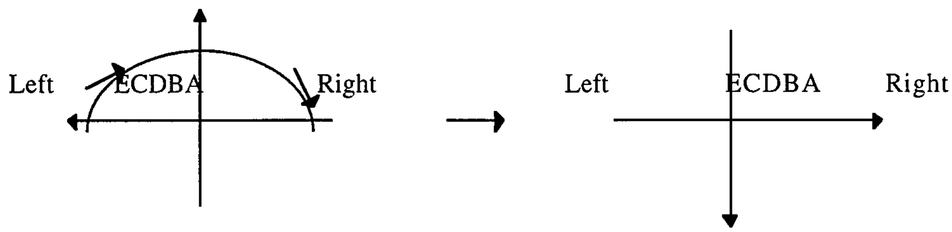
Although the overall effects of direction confirm DeSoto et al.'s predictions, the results obtained for the semicontinuous order (i.e., starting with A) were the exact opposite of those predicted by the authors while the results obtained for the discontinuous order (i.e., starting with E) were exactly as predicted. There are two possible explanations for these results. One arises from the differences in the number of premises (2 vs. 4), the relational terms (nonspatial vs. spatial), and the levels of referential continuity (semicontinuous vs. semicontinuous and discontinuous) that were used respectively in DeSoto et al.'s experiment and in the present experiment.

Another explanation stems from the possibility that subjects were assigning an origin (either A or E) to the spatial reference frames, and that the origin specified the ease of processing the directions. When layouts were described from the object A of the layouts (in the semicontinuous order), subjects found it easier to construct mental models from right to left along the horizontal axis, and from bottom to top along the vertical axis, rather than in the opposite directions. According to Logan's theory (1995), the ease of building mental models from bottom to top suggests that subjects rotated their spatial reference frame 90 degrees clockwise from the horizontal axis in order to align it with the vertical axis using the same origin. For example:



The effect of direction thus rotated with the spatial reference frame. For the object A, mental models were then easier to construct from bottom to top of the vertical axis than from the opposite direction.

When layouts were described from the object E (in the discontinuous order), the effects of direction were reversed. From the object E, mental models were thus significantly easier to construct from left to right along the horizontal axis, and top to bottom along the vertical axis rather than in the opposite directions. Logan's theory suggests that subjects reversed their spatial reference frame 180 degrees from the origin A to project it onto the origin E. An example of this process for the horizontal axis is illustrated below:



Although origin did not constitute a variable in the experimental design, the above results suggest that subjects rotated their spatial reference frame in different orientations according to the different origins to control spatial attention during the construction of mental models. A subsequent experiment could elucidate the relationships between the effects of origin, direction, and referential continuity. Nonetheless, the consistent effects of direction do suggest that subjects used spatial reference frames in building mental models. For semicontinuous order, subjects found it systematically easier to construct mental models from right to left and bottom to top which suggests that these directions form the basic ones of the spatial reference frame.

5.2.2.3 Subjects' explanations. While verbal reports do not meet the robustness of quantitative data, they provide important supportive information concerning subjects' strategies. In accordance with the questionnaire, subjects explained (a) the way in which they represented the objects described in the premise sets; (b) how they organized and remembered the relative locations of the objects; and (c) the final structure of their representation.

Subjects represented the five objects in three different ways. One subject visualized the images of the objects. The others preferred to remember the words or discovered that the initial of each word facilitated their recall ("I used the first letter of each word, initials are easier than words").

They indicated organizing the location of the objects using (a) a reference point, such as the center of the computer screen, from which to position the objects, and/ or (b) a frame of reference such as an imaginary shelf, a three-dimensional grid, or Cartesian coordinate axes. The subject who visualized the objects located them as if they were at sea ("the tank could float!"). Subjects remembered the objects' locations through (a) rehearsal (e.g., "I repeated the objects' order until I had an F or U shape"), and/ or (b) by creating meaningful scenarios (e.g., "I created meaningful scenarios: a ship carrying guns, a tank bombing a ship").

All subjects indicated constructing layouts of the objects in space. They confirmed this representation to the experimenter by spontaneous drawings or by using their hands to imitate the position of the objects. The spatial layouts had a shape, such as (a) an horizontal or vertical line for the 1D problems; (b) an F shape for 2D problems; and (c) a 3D chemical model for 3D problems.

Complementary to the objective results, subjects' explanations support the Mental Models theory's assumptions regarding the mental representation that underlies spatial reasoning. Subjects went beyond the propositional content of the premises to create an integrated mental model. Our results further clarify the nature of these mental models: their properties are geometrical and visual although the signifiers of these mental models consist of words, letters, and exceptionally, images of the objects.

6. Conclusions

The overall goal of this study was to investigate the effects of logical form and geometrical content on the processes that underlie spatial deductive reasoning. The results indicate that the logical form of an argument as well as its geometrical content affect the difficulty of spatial deductive reasoning.

6.1 Role of Logical Form

We addressed the effects of logical form by varying the continuity and the determinacy of the entities' order in the arguments in order to specify the relative importance of the two factors on the processes of spatial reasoning. We assessed the effects of these factors by comparing the opposite predictions made by Hagert's Formal Rules theory (1985; Hagert & Hansson, 1983, 1984) and Johnson-Laird's Mental Models theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991, 1993). The Formal Rules theory predicted that referential determinacy would be more important than referential continuity in determining the difficulty of spatial reasoning. If so, problems presented in a discontinuous order would be easier than those presented in a semicontinuous order. The Mental Models theory made the opposite prediction: referential continuity among a set of premises would be a necessary and prior condition for the construction of a single mental model. Consequently, problems presented in a discontinuous order should be more difficult than those presented in a semicontinuous one as the former order would interfere with the construction of a mental model.

The results confirmed the Mental Models theory. Deductions from the discontinuous order were significantly more difficult than from the semicontinuous order although the former involved a shorter formal derivation than the latter. In the discontinuous order, problems requiring two independent mental layouts (problem type 3) were also significantly more difficult than those involving two mental layouts consisting of the same entities (problem type 2 in the semicontinuous order) although both problem types involved formal derivations of the same length. In the discontinuous order, problems of type 3 were also more difficult than those of type 1 and type 2. The first three premises of problem type 3 required subjects to hold two independent mental layouts for 27 seconds in working memory. In contrast, the first two premises of problem type 1 and type 2 required them to hold two independent mental layouts for 18 seconds in working memory. Using the third premise, subjects could then integrate these mental layouts into a single one. Given the problems' temporal parameters, the results suggest that the third premise of the set of four was critical in determining the ease with which subjects were able to integrate the mental layouts into a mental model. In the discontinuous order, the premises of problem type 3 were thus more likely to interfere with the construction of yet a single mental model than those of problem type 1 and type 2. The results thus show that referential discontinuity among premises has precedence over referential indeterminacy in determining the difficult of spatial reasoning.

The Formal Rules theory and the Mental Models theory also made opposite predictions regarding the effects of referential determinacy. The Formal Rules theory predicted that determinate orders in which all entities are involved in a formal derivation would be more difficult than indeterminate orders despite the fact that the former would yield only one mental model and the latter two mental models. The Mental Models theory made the converse prediction. The semicontinuous order provided an independent measure of the effects of referential determinacy. The results showed that problems based on two mental models (problem type 2) were significantly more difficult than problems based on one mental model (problem type 3) although the former involved a shorter formal derivation than the latter. These results generalized across all three dimensional conditions. When referential continuity allowed the continuous integration of a mental model, referential indeterminacy became the critical factor in determining the difficulty of reasoning. The effects of referential determinacy, together with those of referential continuity, thus corroborate the Mental Models theory's principle predictions which generalise across all three dimensional conditions.

6.2 Role of Geometrical Content

We addressed the role of geometrical content by varying the euclidian (number of dimensions) and the projective relations (orientation and direction) among the entities. We elucidated the effects of these variables in view of the opposing predictions made by the Formal Rules theory and the Mental Models theory of spatial deduction. We also investigated the effects of geometrical content in light of the Spatial Reference Frame theory to specify whether the mental representations of geometrical relations are structured relative to spatial reference frames, i.e., according to a set of three coordinate axes which have an origin, orientation, and direction.

The geometrical content of the arguments systematically affected the difficulty of spatial reasoning although the formal derivations were of the same length across contents, and although the ED relation involved a longer formal derivation than the EB relation. Thus, the difficulty of spatial deductions increased systematically with the number of dimensions that subjects had to integrate and inspect within a mental model. These results suggest that mental models reproduce euclidian relations among entities in a way that is similar to which mental images reproduce metric relations (Kosslyn et al. 1978). This property suggests a functional relationship between constructing mental models in 1D, 2D and 3D, scanning mental images of such models, and perceptually exploring physical models of 1D, 2D and 3D spatial relations. Although this hypothesis remains to be investigated, it posits a continuum between the process of perceiving geometrical relations, visually imagining such relations, and constructing mental models of the same relations. This continuum would be based on the partial isomorphisms than exists between perceptual structures and logical structures (Piaget & Morf, 1958).

The projective relations also reliably affected the difficulty of spatial reasoning. Three-dimensional mental models were systematically more difficult to construct from vertical layouts than from horizontal ones indicating that the line of sight axis was easier to access relative to the horizontal axis than to the vertical axis. As predicted by the Content-Specific Rules theory (DeSoto et al., 1965), subjects also showed systematic patterns of directional preferences in constructing mental models in one set of directions rather than in the opposite one. The effects of the projective relations indicate that the subjects' spatial reference frames have an origin, orientation, and direction (Logan, 1995) that direct conceptual attention during the construction of mental models. Logan's spatial reference frame hypothesis thus generalizes to the process of spatial deductive reasoning while providing a complementary account of the effects of geometrical content which confirm the Mental Models theory.

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9. Appendix A

Appendix A

The appendix contains the 144 problems used in the experiment. It describes the variables used to create the problems, namely logical form [(referential determinacy (3), referential continuity (2)] and geometrical content [dimensional condition (3), orientation (2), direction (2)], and question types (number of dimensions). The difficulty of the problems yields two sets of predictions, one based on the number of mental models and the other on the number of inferential steps in a formal derivation.

Logical form		Geometrical content				Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
Referential determinacy: problem type 1		Dimensional Condition 1D Horizontal layouts and directions		Vertical layouts and directions		EB relation 1D	ED relation 1D		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
Referential continuity		H1	H2	V1	V2	Examples				
Semi-continuous order	A r B C r B D r B C r E	right left directly left directly right	left right directly right directly left	below above directly above directly below	above below directly below directly above	E left of B?	E left of D?	1	2 Steps 1: r(CE) <--> - r(EC) 2: r(EC) & r(CB) => r(EB)	4 Steps 1: r(CE) <--> -r(EC) 2: r(EC) & r(CB) => r(EB) 3: r(DB) <--> -r(BD) 4: r(EB) & r(BD) => r(ED)
Dis-continuous order	E r C B r D C r B A r B	directly left directly right left right	directly right directly left right left	directly above directly below above below	directly below directly above below above	E left of B?	E left of D?	1 (1) (note 2)	1 Step 1: r(EC) & r(CB) => r(EB)	2 Steps 1: r(EC) & r(CB) => r(EB) 2: r(EB) & r(BD) => r(ED)
Mental layouts		ECDBA	ABDCE	E C D B A	A B D C E					

(Appendix A continues)

Logical form		Geometrical content				Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
Referential determinacy: problem type 2		Dimensional condition 1D Horizontal layouts and directions		Vertical layouts and directions		EB relation 1D	ED relation 1D		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
Referential continuity		H1	H2	V1	V2	Examples	Examples			
Semi-continuous order	B r A C r B D r B C r E	right left directly left directly right	left right directly right directly left	below above directly above directly below	above below directly below directly above	E left of B?	E left of D?	2	2 Steps 1: r(CE) <--> - r(EC) 2: r(EC) & r(CB) => r(EB)	4 Steps 1: r(CE) <--> -r(EC) 2: r(EC) & r(CB) => r(EB) 3: r(DB) <--> -r(BD) 4: r(EB) & r(BD) => r(ED)
Dis-continuous order	E r C B r D C r B B r A	directly left directly right left right	directly right directly left right left	directly above directly below above below	directly below directly above below above	E left of B?	E left of D?	2 (1)	1 Step 1: r(EC) & r(CB) => r(EB)	2 Steps 1: r(EC) & r(CB) => r(EB) 2: r(EB) & r(BD) => r(ED)
Mental layouts		ECADB AECDB	BDACE BDCEA	E A C E A C D D B B	B B D D A C C E E A					

(Appendix A continues)

Logical form		Geometrical content				Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
Referential determinacy: problem type 2		Dimensional Condition 2D Horizontal layouts and directions		Vertical layouts and directions		EB relation 2D	ED relation 1D		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
Referential continuity		H1	H2	V1	V2	Examples	Examples			
Semi-continuous order	B r A C r B D r B C r E	right left directly below directly above	left right directly above directly below	below above directly left directly right	above below directly right directly left	E left of B? E below B?	E left of D?	2	2 Steps 1: r(CE) <--> - r(EC) 2: r(EC) & r(CB) => r(EB)	4 Steps 1: r(CE) <--> -r(EC) 2: r(EC) & r(CB) => r(EB) 3: r(DB) <--> -r(BD) 4: r(EB) & r(BD) => r(ED)
Dis-continuous order	E r C B r D C r B B r A	directly below directly above left right	directly above directly below right left	directly left directly right above below	directly right directly left below above	E left of B? E below B?	E left of D?	2(1)	1 Step 1: r(EC) & r(CB) => r(EB)	2 Steps 1: r(EC) & r(CB) => r(EB) 2: r(EB) & r(BD) => r(ED)
Mental layouts		CAB E D ACB ED	D E BAC DE BCA	EC A DB A EC DB	BD A CE BD CE A					

(Appendix A continues)

Logical form		Geometrical content				Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
Referential determinacy: problem type 2		Dimensional Condition 3D Horizontal layouts and directions		Vertical layouts and directions		EB relation 2D	ED relation 3D		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
Referential continuity		H1	H2	V1	V2	Examples	Examples			
Semi-continuous order	B r A C r B D r B C r E	right left directly in front directly above	left right directly behind directly below	below above directly in front directly right	above below directly behind directly left	E left of B? E below B?	E left of D? E below D? E behind D?	2	2 Steps 1: r(CE) <--> - r(EC) 2: r(EC) & r(CB) => r(EB)	4 Steps 1: r(CE) <--> -r(EC) 2: r(EC) & r(CB) => r(EB) 3: r(DB) <--> -r(BD) 4: r(EB) & r(BD) => r(ED)
Dis-continuous order	E r C B r D C r B B r A	directly below directly behind left right	directly above directly in front right left	directly left directly behind above below	directly right directly in front below above	E left of B? E below B?	E left of D? E below D? E behind D?	2(1)	1 Step 1: r(EC) & r(CB) => r(EB)	2 Steps 1: r(EC) & r(CB) => r(EB) 2: r(EB) & r(BD) => r(ED)
Mental layouts		C A B/D E A C B/D E	E B/D C A E B/D A C	C E A A C E B/D B/D	B/D B/D A C E C E A					

(Appendix A continues)

Logical form		Geometrical content				Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
Referential determinacy: problem type 3		Dimensional condition 1D Horizontal layouts and directions		Vertical layouts and directions		EB relation 1D	ED relation 1D		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
Referential continuity		H1	H2	V1	V2	Examples	Examples			
Semi-continuous order	A r B C r B D r A C r E	right left directly left directly right	left right directly right directly left	below above directly above directly below	above below directly below directly above	E left of B?	E left of D?	1	2 Steps 1: r(CE) <--> - r(EC) 2: r(EC) & r(CB) => r(EB)	6 Steps 1: r(CE) <--> -r(EC) 2: r(EC) & r(CB) => r(EB) 3: r(AB) <--> -r(BA) 4: r(EB) & r(BA) => r(EA) 5: r(DA) <--> -r(AD) 6: r(EA) & r(AD => r(ED)
Dis-continuous order	E r C A r D C r B A r B	directly left directly right left right	directly right directly left right left	directly above directly below above below	directly below directly above below above	E left of B?	E left of D?	1 (2)	1 Step 1: r(EC) & r(CB) => r(EB)	4 Steps 1: r(EC) & r(CB) => r(EB) 2: r(AB) <--> -r(BA) 3: r(EB) & -r(BA) => r(EA) 4: r(EA) & r(A D) => r(ED)
Mental layouts		ECBDA	ADBCE	E C B D A	A D B C E					

(Appendix A continues)

Logical form		Geometrical content				Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
Referential determinacy: problem type 3		Dimensional condition 2D Horizontal layouts and directions		Vertical layouts and directions		EB relation 2D	ED relation 1D		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
Referential continuity		H1	H2	V1	V2	Examples	Examples			
Semi-continuous order	A r B C r B D r A C r E	right left directly below directly above	left right directly above directly below	below above directly left directly right	above below directly right directly left	E left of B? E below B?	E left of D?	1	2 Steps 1: r(CE) <--> - r(EC) 2: r(EC) & r(CB) => r(EB)	6 Steps 1: r(CE) <--> -r(EC) 2: r(EC) & r(CB) => r(EB) 3: r(AB) <--> -r(BA) 4: r(EB) & r(BA) => r(EA) 5: r(DA) <--> -r(AD) 6: r(EA) & r(AD => r(ED)
Dis-continuous order	E r C A r D C r B A r B	directly below directly above left right	directly above directly below right left	directly left directly right above below	directly right directly left below above	E left of B? E below B?	E left of D?	1 (2)	1 Step 1: r(EC) & r(CB) => r(EB)	4 Steps 1: r(EC) & r(CB) => r(EB) 2: r(AB) <--> -r(BA) 3: r(EB) & -r(BA) => r(EA) 4: r(EA) & r(A D) => r(ED)
Mental layouts		C B A E D	D E A B C	E C B D A	A D B C E					

(Appendix A continues)

Logical form		Geometrical content				Question type & number of dimensions requested in question type		Number mental models	Inferential steps in the formal derivation of the EB and ED relations	
Referential determinacy: problem type 3		Dimensional Condition 3D Horizontal layouts and directions		Vertical layouts and directions		EB relation 2D	ED relation 3D		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
Referential continuity		H1	H2	V1	V2	Examples	Examples			
Semi-continuous order	A r B C r B D r A C r E	right left directly in front directly above	left right directly behind directly below	below above directly in front directly right	above below directly behind directly left	E left of B? E below B?	E left of D? E below D? E behind D?	1	2 Steps 1: r(CE) <--> - r(EC) 2: r(EC) & r(CB) => r(EB)	6 Steps 1: r(CE) <--> -r(EC) 2: r(EC) & r(CB) => r(EB) 3: r(AB) <--> -r(BA) 4: r(EB) & r(BA) => r(EA) 5: r(DA) <--> -r(AD) 6: r(EA) & r(AD => r(ED)
Dis-continuous	E r C A r D C r B A r B	directly below directly behind left right	directly above directly in front right left	directly left directly behind above below	directly right directly in front below above	E left of B? E below B?	E left of D? E below D? E behind D?	1(2)	1 Step 1: r(EC) & r(CB) => r(EB)	4 Steps 1: r(EC) & r(CB) => r(EB) 2: r(AB) <--> -r(BA) 3: r(EB) & -r(BA) => r(EA) 4: r(EA) & r(A D) => r(ED)
Mental layouts		C B A/D E	E A/D B C	E C B A/D	A/D B C E					

Note 1. The layouts depicted in each column are identical for the semicontinuous and discontinuous orders.

Note 2. In the discontinuous order, the first three premises of problem type 3 have no referent in common thus requiring two independent mental layouts (indicated in parentheses) to be held in working memory. For problems of type 1 and type 2, the first two premises yield two independent mental layouts which can be integrated in one mental layout using the third premise (indicated in parentheses). In the semicontinuous order, the referential continuity among the first three premises allows the continuous construction of a mental model.

Note 3. The letter "r" represents a relation described in a premise or deduced from an inferential step. The sign -r represents the inverse of a relation specified between an object pair, such as -right(BA) is equivalent to left(AB). The symbol <--> and => denote a relation of equivalence and implication respectively.

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In air navigation, pilots must mentally represent the relative position of aircraft in a three dimensions. They must also deduce from their mental representation the relative position of other aircraft which has not been explicitly specified in any of the information that they have received. Spatial deductive reasoning is critical for both the pilot's safety and the accomplishment of his mission. It is a difficult aspect of formal logic which is particular prone to errors especially under intense mental workload. Despite the importance of spatial deductive reasoning, the processes that underlie this logical activity are still unclear as scientists have proposed opponent theories of formal logic to account for these processes.

The overall goal of this study is to investigate the effects of logical form and geometrical content on spatial deductive reasoning by comparing two opponent theories of deductive reasoning: Hagert's Formal Rules theory and Johnson-Laird's Mental Models theory. Our second goal is to specify, through the effects of geometrical content, how humans structure their mental representation of geometrical relations, and if they do so relative to spatial reference frames. We will address this issue in view of the Content-Specific Rules theory and the Spatial Reference Frame theory.

Twenty-six subjects solved 144 spatial deductive problems which differed by their logical form and geometrical content. We addressed the effects of logical form by varying the continuity and the determinacy of the entities' order in the arguments. The logical form of the arguments allowed comparison of pairs of problems having either (a) formal derivations of equal length but different numbers of mental models, and (b) formal derivations of different lengths but equal numbers of mental models. We addressed the effects of geometrical content by varying the euclidian (number of dimensions) and projective relations (orientation and direction) among the entities. The number of mental models and the number of steps in the formal derivations of valid conclusions were the same across geometrical contents. Overall, the effects of logical form and geometrical content confirmed the Mental Models theory's predictions while refuting those of the Formal Rules theory. The effects of geometrical content suggest that subjects constructed mental models which reproduce the geometrical relations among entities relative to spatial reference frames. This study will provide an experimental basis from which to measure spatial reasoning in pilots, and help design visualization techniques to train air-to-air basic flight maneuvers.

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Spatial Reasoning

Formal Rules theory

Mental Models theory

Spatial relations

Referential Continuity

Referential Determinacy

Spatial Reference Frames